A comparison of Gompertz and logistic growth models using post-natal

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Abstract – Birth weights and subsequent weights of babies play vital role in early childhood development and have important effect on later lives as adults. It also presented a model to explain the growth trajectory of infants monitored in seven health facilities providing post-natal services in Bolgatanga Municipality.Records of three hundred weights of babies were taken from January to December 2013 in these facilities for the study. Sixty five of these records were left out due to the inability of the nursing mothers to attend at most six post-natal cares within the year. The available records of number of children qualified for the analysis were 235 making up 3055 observations. However, later months exhibited high level of missing data which we used multiple imputation method to provide reasonable data points to replace the missingness. Gompertz and logistic growth models were used to fit the data and both models fitted the data well and could be used for prediction. Both Gompertz and Logistic growth models proved a significant increase pattern in the growth of babies and estimating the maximum growth rate at 1.517 and 4.59 kg respectively for a month with high reliability and variability in monitoring the growth rate of babies. The general growth pattern of the baby weight disclosed that while Gompertz model showed an increasing exponential growth pattern and is asymmetrical about inflection, Logistic growth pattern showed a function which is increasing in a quadratic growth pattern of mean weights of babies in the Bolgatanga Municipality overtime with high level of significance of all the parameters in the model at 5% significant level and is symmetrical about the inflection. Prediction of baby weight with time as measure of growth is to be done with high accuracy and precision. Even-though the two growth models have approximately the same high degree of variability indicating high reliability in their prediction of baby weights, Gompertz model proved to be relatively better because it gave the least values of root mean square error and residual deviance. Hence Gompertz model was selected as the better model for prediction of the mean change of weights of babies over time in the municipality with the growth equation

weight=9.688 exp(-exp(0.221-0.462t))and increasing at a growth rate of d(weight)/d(t) = 0.462y exp (0.221 - 0.462t) with the maximum achievable weight increment per month as 1.517kg.Finally, all the weights of babies that we considered in our study established that growth of weights of babies studied is not linear but indicated non-linear growth pattern of Gompertz and/or logistic over time.

From the study conducted, Gompertz model and the reduced mixed model are more suitable for explaining the mean weights of babies in the municipality overtime.The Gompertz model equation is given as:

weight=9.688 exp(-exp(-0.221(x-0.462)))

Whilst the reduced model equation is given as:

$\hat{Y} = 1.1259w_1+ 0.5412w_2+ 0.3185w_3+0.3724w_4-0.1707w_5$

Keywords: Gompertz, logistic, missingness, multiple imputations, post-natal

1.0 Introduction

Studying the growth pattern of natural and social phenomena has become increasingly important to researchers because it provides means for visualizing growth patterns over time, and the equations gotten help in making reasonable predictions of expected growth rate of these phenomena at a specific instant of time thus indicating permanent increase in the size of any object of interest to researchers (Tzeng& Becker, 1981; Yakupoglu&Atil, 2001; Sengul&Kiraz, 2005).

Nonlinear regression is an extended linear regression technique in which a nonlinear mathematical model is used to describe the relationship between the response variable and the predictor variables (Bales and Watts 1988). A nonlinear regression model is a model that contains at least one of the parameters in a nonlinear form.

Growth which is a permanent increase in size over time was modeled as an exponential function. The earlier of such models were the Malthusian exponential population models (Malthus, 1798). Exponential models had limitations due to the fact that things which are capable of growing do so within certain bound but the Malthusian models give an impression that growth is boundless which is perfectly true in mathematical or theoretical sense but deviates from growth of organisms. To overcome the limitations of the exponential models, logistic model is proposed (Chasnov, 2009) which included the restriction required to make accurate prediction of growth has been studied and used to model growth in several fields of study such as agriculture, biology, economics, physics, finance and chemistry. Pearl and Reed (1920) modeled growth with logistic curve.

The Gompertz growth curve has also been studied and used extensively in phenomena in the actuarial science. Comparing the characteristics of logistic and Gompertz growth models was presented in a paper by Winsor (Winsor, 1932) and he concluded that neither in reality seemed superior in terms of accuracy in prediction of growth at a certain point in time.

Wright (1926) first used the Gompertz curve in modeling biological growth. Also, Wright and Davidson (1928) carried out studies where they used the Gompertz curve to illustrate the growth in body weight of cattle. The growth and development of children between 1997 and 2003 the world over was assessed using growth curves which was in response to World Health Organization under the Multicentre Growth Reference Study (MGRS).They combined a longitudinal follow-up from the time of birth to age 24 months as well as a cross-sectional survey of children between the age bracket of 18 to 71 months. A total of 8440 primary growth data and related information were gathered for the analysis. The survey was basically designed to come out with a standard by choosing healthy children from mothers who used health-promoting practices living under conditions likely to favor the achievement of their full genetic growth potential (WHO, 2006).

In Ghana, a child born either at home or in a health facility is mandated to have a postnatal care (care given to the mother and the child one month after birth) and subsequent monthly weighing of the baby known as child welfare care. The reasons for the child welfare care include: growth monitoring of the child, for early detection of any abnormal growth existing condition for referral to the appropriate health unit for special attention, for immunization against the six childhood killer diseases and for counseling of the mothers with regards to the welfare of both the child and the nursing mother.

The first month of life, called the newborn or neonatal period is the riskiest period in the life of every individual because whilst inside their mothers, babies are safe, warm and well fed.

After birth, newborns have to adapt to a different way of feeding, breathing and staying warm otherwise they can get sick easily leading to death. Thus, the health and survival of newborn children depend largely on their weights and how they are cared for. Out of every 100 children born alive, about 10 die before reaching the age of five years (WHO, 2012).

Statistics of Ghana Demographic Health Survey (GDHS,2005) and Multiple Indicator Cluster Survey (MICS, 2006) suggest that child mortality is increasing with a shocking drop in performance by the Upper East Region even though the country tries hard to reduce under-five Mortality Rate of the Millennium Development Goals 4 and 5 by 2015.

A five year child survival program known as Essential Newborn Care started in 2012 in Bolgatanga Municipality as well as the other municipalities and districts within the region is designed to improve the birth, soon after birth and the postnatal period (UNICEF, 2013).
The health and survival of newborn children depend largely on how they are cared for. Recent worldwide evaluation points to the fact that commitment to raising the standards of the health status of newborn babies' yield meaningful socio-economic contributions (Yinger and Ransom, 2003).

Further research on newborn babies revealed that child survival programme has assisted in the reduction of death rates among under-five year old babies over the past twenty five 25 years and the greatest impact has been on reducing mortality from diseases which attack infants and children over one month old. Hence, huge proportions of infant mortality take place between the first month of life (the neonatal period), a period when a child's risk of death is almost fifteen (15) times greater than at any other time before the first birthday (Yinger and Ransom, 2003).

Tinker and Ransom (2003) stipulated that, though newborn health is closely related to that of their mothers, newborns have a unique need that must be addressed in the content of maternal and child health services. Their further argument was that millions of newborn deaths could be avoided if more resources were invested in proven low-cost interventions designed to address newborn needs.

A similar research on newborn babies estimated that almost two-thirds of infant mortality occurs in the first month of life, of which more than two-thirds die in their first week, and among them, two-thirds die in their first 24 hours (Lawn, 2001).

2.0 Research Methods

2.1Data used

It also presented a model to explain the growth trajectory of infants monitored in seven health facilities providing post-natal services in Bolgatanga Municipality.

Records of three hundred weights of babies were taken from January to December 2013 in these facilities for the study. Sixty five of these records were left out due to the inability of the nursing mothers to attend at most six post-natal cares within the year.

The available records of number of children qualified for the analysis were 235 making up 3055 observations.

However, later months exhibited high level of missing data which we used multiple imputation method to provide reasonable data points to replace the missingness.

2.2Growth models

Apart from using the profile analysis to examine and assess repeated measurements which assumes a linear model framework we can also assess growth models using the non-linear approach. The most popular of them are the Gompertz and logistic models (Winsor, 1932).

2.2.1Gompertz growth model

These models can have three or four parametric models but in this study we used the three parameter growth models in our analysis.

The Gompertz 3-parametric growth model is given by;

$$y = ke^{-e^{-\alpha(x-\beta)}} \quad (1)$$

We can write this as

$$y = ke^{-e^{(\mu-\alpha x)}} \qquad (2)$$

where $\mu = \alpha\beta$, y = mean weights of babies and x = time in months

2.2.2Growth rate of mean weights of babies

To find $\frac{dy}{dx}$ from (2),

we used the chain rule by

letting

$$t = -e^{(\mu-\alpha x)} \qquad (3)$$

and

$y = ke^t$ (4)From (3),

$\frac{dt}{dx} = -\alpha(-e^{(\mu-\alpha x)})$

$= \alpha e^{(\mu-\alpha x)}$ (5)

and from (5),

$$\frac{dy}{dt} = ke^t \,(6)$$

$\frac{dy}{dx} = \frac{dt}{dx}.\frac{dy}{dt} = ke^t(\alpha e^{(\mu-\alpha x)}) = \alpha y e^{(\mu-\alpha x)}$ (7)

From (2), we divided both sides by k and took logarithms of both sides and got

$\log\left(\frac{y}{k}\right) = -e^{(\mu-\alpha x)}$ (8)Dividing through by -1 implies

$$\log\left(\frac{k}{y}\right) = e^{(\mu-\alpha x)} \qquad (9)$$

Substituting (9) into (7) implies

$$\frac{dy}{dx} = \alpha y e^{(\mu-\alpha x)} = \alpha y \log\left(\frac{k}{y}\right) \qquad (10)$$

From (10), the relative growth rate of mean weights of babies as a function of time is :

$$\frac{1}{y}\frac{dy}{dx} = \alpha e^{(\mu-\alpha x)} \qquad (11)$$

Also from (10),the relative growth rate of mean weights of babies as a function of size is:

$$\frac{1}{y}\frac{dy}{dx} = \alpha(\text{logk} - \log y) \qquad (12)$$

To find the $x -$ coordinate, we equated (10) to zero i.e.

$$\alpha y e^{(\mu-\alpha x)} = 0 \qquad (13)$$

Solving (13) gives as

$$x = \frac{\mu}{\alpha}$$

When put into (2) yields

$$y = \frac{k}{e}$$

Testing for nature of stationary points from (10),

$$\frac{dy}{dx} = \alpha y e^{(\mu - \alpha x)}$$

Using the product rule, let

$$m = y = ke^{-e^{(\mu - \alpha x)}}$$

and

$$n = \alpha e^{(\mu - \alpha x)}$$

Then, $\frac{dm}{dx} = \alpha y e^{(\mu - \alpha x)}$

And

$\frac{dn}{dx} = -\alpha^2 e^{(\mu - \alpha x)}$(14)
but

$$\frac{d^2y}{dx^2} = m\frac{dn}{dx} + n\frac{dm}{dx} = y\left(-\alpha^2 e^{(\mu-\alpha x)}\right) + \alpha e^{(\mu-\alpha x)}\left(\alpha y e^{(\mu-\alpha x)}\right) = \alpha^2 y e^{(\mu-\alpha x)}\left(e^{(\mu-\alpha x)} - 1\right) \qquad (15)$$

To test for the nature of stationary points, we put $x = \frac{\mu}{\alpha}$ into (15) i.e.

$$\frac{d^2y}{dx^2} = \alpha^2 y e^{(\mu-\alpha x)}\left(e^{\left(\mu - \alpha\left(\frac{\mu}{\alpha}\right)\right)} - 1\right) = 0$$

Since

$$\frac{d^2y}{dx^2} = 0$$

the point of inflexion of mean weights of babies occurred at $x = \frac{\mu}{\alpha}$.
The ordinate at the point of inflexion is $y = \frac{k}{e}$ .The maximum growth rate of mean weights of babies in the municipality from (10) is

$\frac{dy}{dx} = \alpha y e^{(\mu - \alpha x)}$. But $y = \frac{k}{e}$ and $x = \frac{\mu}{\alpha}$ when put into (10) gave a maximum growth rate of mean weights of babies in the municipality as: $\left(\frac{dy}{dx}\right) = \frac{\mu k}{e}$, asymptotes occurred at y = 0 and y = k

From practice, we fit Gompertz growth model if the point of inflexion is about 35-40% of the total growth attained.

2.2.4 Logistic growth model
We can also fit a logistic growth model for our data which is a close counterpart of the Gompertz model. The logistic model is given by;

$$y = \frac{k}{\left(1 + e^{-\alpha(x-\beta)}\right)} \qquad (16)$$

which can be rewritten as

$$y = \frac{k}{1 + e^{(\mu - \alpha x)}} =$$

$$k(1 + e^{(\mu - \alpha x)})^{-1} \quad (17)$$

where

$$\mu = \alpha\beta$$

2.2.5Growth rate of mean weights of babies

To find $\frac{dy}{dx}$, from (17) we used the chain rule by letting

$t = 1 + e^{(\mu - \alpha x)}$ (18)

and

$$y = kt^{-1} \quad (19)$$

From (3),

$\frac{dt}{dx} = -\alpha e^{(\mu - \alpha x)}$ (20)

Also from

$$\frac{dy}{dt} = -kt^{-2} \quad (21)$$

but

$\frac{dy}{dx} = \frac{dt}{dx} \cdot \frac{dy}{dt} = -kt^{-2}\left(-\alpha e^{(\mu - \alpha x)}\right)$

$$= k\alpha e^{(\mu - \alpha x)} t^{-2} \quad (22)$$

From (16),

$e^{(\mu - \alpha x)} = \frac{k - y}{y}$ (23)

Substituting(23)into(22)gives

$$\frac{dy}{dx} = \frac{\alpha y}{k}(k - y) \quad (24)$$

The maximum growth rate of mean weights of babies is$\frac{\alpha k}{4}$, asymptotes occurred at y = 0 and y = k Point of inflexion occurred at $x = \frac{\mu}{\alpha}$ and$\frac{k}{2}$.

The relative growth of mean weights of babies as a function of time is given by:

$\frac{1}{y}\frac{dy}{dx} = \frac{\alpha}{1 + e^{-\mu + \alpha x}}$(25)

The relative growth rate of mean weights of babies as a function of size is given by:

$$\frac{1}{y}\frac{dy}{dx} = \frac{\alpha}{k}(k - y)$$

The principal interest in the two models is to compare them and choose the one which is superior in terms of its ability to provide prediction with the least errors

3.0Results and Discussions

3.1Growth model

3-Parameter Gompertz and 3-Parameter Logistic models are the growth models used in this study. Their respective figures and tables are displayed below with b1, b2 and b3 representing α, β and γ respectively
3.1.1Gompertz growth model
Figure 4.9 shows the pattern of mean change in weights of babies over time using 3-Parameter Gompertz growth model

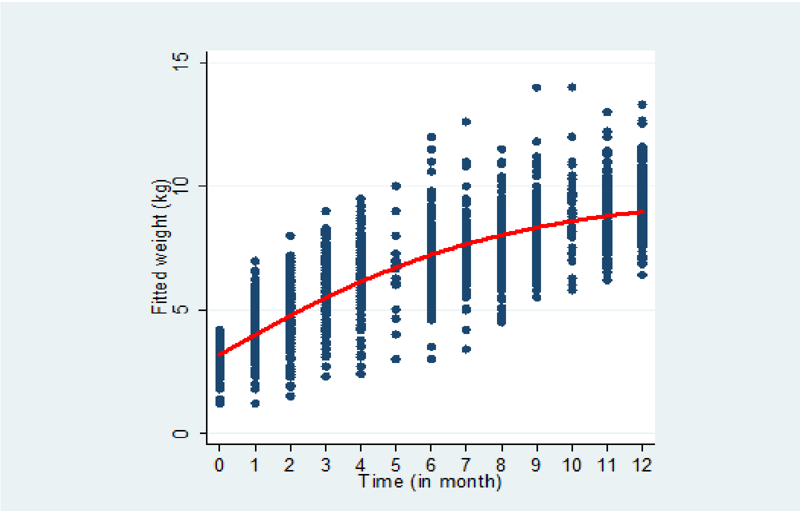

Figure 1.0Gompertzgrowth of mean weights overtime

Figure 1.0 shows that the general pattern of change in mean weights of babies increases over time. The Gompertz growth curve of the mean weights of babies over a one year period asymptotes occur at $y=0$ and $y=k$ with the point of inflection showing at $y=k/e$ and $t=\alpha/\beta$ with a maximum growth rate of $\beta k/e$
Table 4.10 below shows the three parameter Gompertz growth model of baby weights over time

Table 1: 3-Parameter Gompertz growth model of baby weights

3-parameter Gompertz function k*exp(-exp(α *(time-β)))

Weight	Estimate	Standard Error	*t*	□ > □	[95% conf. interval]
▱	9.6880	0.1140	84.3900	0.0000	9.4630
▱	0.2210	0.0080	27.0200	0.0000	0.2050
▱	0.4620	0.0610	7.5100	0.0000	0.3410

Therefore, the fitted Gompertz growth model from table 1.0 is stated as shown below:

$$weight = 9.688\exp〖(-\exp(-0.221(x-0.462)))〗 \quad (26)$$

Where $x=0,1,2,3,\ldots\ldots x\_n$ (i.e.x=time in months)

The rate of change in mean weights of babies is then given as:
(d(weight))/(d(x))=0.2210y exp(0.1021-0.2210x) (27)
Relative growth rate as a function of weight:
1/y (d(weight))/(d(x)) = 0.2210(0.9862-logy) (28)
Relative growth rate as a function of time:
1/y (d(weight))/(d(x)) =0.2210e^((0.1021-0.2210x)) (29)
Maximum growth rate of mean weights of babies = (μk)/e=0.102(9.688)/2.72=0.3633 = 36.33%
Asymptotes occurred at y=0 and y=9.688 and the point of inflexion occurred at the point (0.46, 3.56)
3.1.2Logistic growth model
Figure 2.0 shows the pattern of mean change in weights of babies over time using 3-Parameter Logistic growth model

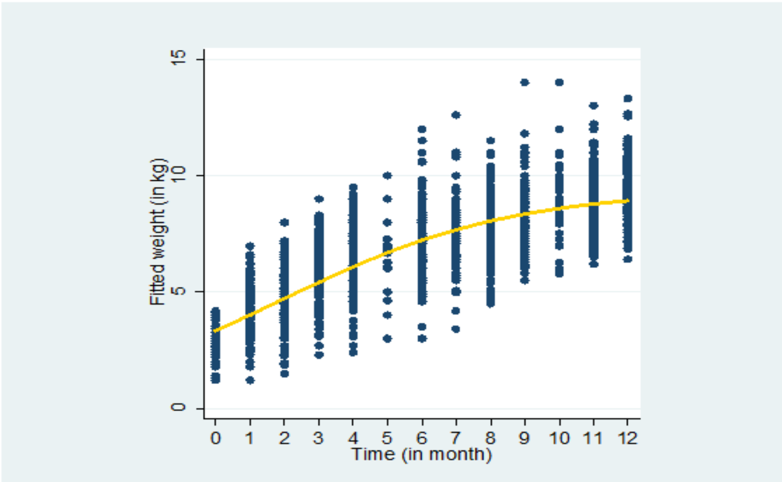

Figure 1: Logistic growth pattern of mean baby weights overtime

That of the logistic growth curve model has the asymptotes showing at
y=0 and
y=k.
With the point of inflection at y=k/2 and t=α/β
Table 2 shows the three parameter logistic growth model of baby weights over time.

Table 2: 3-Parameter Logistic growth model of weights of babies

3-parameter logistic function weight_=□ /(1+exp(-□ (t-□)))						
Weight	Estimate	Standard Error	□	□ > □	[95%conf. interval]	
□	9.3410	2.0500	106.8700	0.0000	9.1700	9.5130
□	0.3020	0.0090	32.0090	0.0000	0.2840	0.3210
□	1.926	0.0770	24.0900	0.0000	1.7740	2.0780

The fitted 3-Parameter Logistic model is shown below:

$$\square\square\square\square h\square = \frac{9.341}{(1+\square\square\square(-0.302(\square-1.926)))}$$ Where $\square = 0,1,2,3,\ldots\ldots\square_{\square}$ (i.e.$\square$ = time in months)

The rate of change in mean weights of babies is then given as; $\frac{\square(\square\square\square\square ht)}{d(x)} = 0.0323y(9.3410-y)$ $=0.3017y - 0.0323y^2$ (30)

Relative growth rate as function of weight: $\frac{1}{y}\frac{d(weight)}{d(x)} = 0.0323(9.341-y)$ (31)

Relative growth rate as function of time: $\frac{1}{y}\frac{d(weight)}{d(x)} = \frac{0.3020}{1+e^{0.3020x-9.341}}$ (32)

Maximum growth rate= $\frac{\mu k}{4} = \frac{0.5816(9.341)}{4} = 1.358 = 135.8\%$

Asymptotes occurred at $y = 0$ and $y = 9.341$ and the point of inflexion occurred at the point (0.46, 4.67)

Where $\mu = \alpha\beta$

Hence the point of inflection for this growth model is (0.157, 4.671)

Thus at 5% significant level, the logistic growth model indicated that there was a significant increase in weight with time but showing inflection at (0.157, 4.671). The optimum increase in the weight of the babies from the logistic growth pattern was 4.59.

3.4 Growth Model comparison

Gompertz and Logistic models are compared using the model selection criteria as shown below.

Table 3: Model selection criteria for Gompertz and Logistic models

Model	R-squared	Adjusted R-squared	RMSE	Residual deviance
Gompertz	0.9740	0.9740	1.1350	9442.0630
Logistic	0.9730	0.9730	1.1460	9503.5630

Table 3 gives the model selection criteria of Gompertz and logistic models. The Gompertz model had lower root mean square error of 1.135, residual deviance of 9442.063 and equal higher values of R-squared and Adjusted R-squared of 97.4% compared to the Logistic model with higher values of root mean square error of 1.146 and residual error of 9503.563 and equal lower values of R-squared and Adjusted R-squared of 97.3%. Since the Gompertz model gives the least values of root mean square error and residual deviance, it is selected as the better model for prediction of the mean change of weights of babies over time as compared to the logistic model.

Conclusion

Both Gompertz and Logistic growth models proved a significant increase pattern in the growth of babies and estimating the maximum growth rate at 1.517 and 4.59 kg respectively for a month with high reliability and variability in monitoring the growth rate of babies. The general growth pattern of the baby weight disclosed that while Gompertz model showed an increasing exponential growth pattern and is asymmetrical about inflection, Logistic growth pattern showed a function which is increasing in a quadratic growth pattern of mean weights of babies in the Bolgatanga Municipality overtime with high level of significance of all the parameters in the model at 5% significant level and is symmetrical about the inflection. Prediction of baby weight with time as measure of growth is to be done with high accuracy and precision. Even-though the two growth models have approximately the same high degree of variability indicating high reliability it their prediction of baby weights, Gompertz model proved to be relatively better because it gave the least values of root mean square error and residual deviance.

Hence Gompertz model was selected as the better model for prediction of the mean change of weights of babies over time in the municipality with the growth equation

weight=9.688 exp(-exp(0.221-0.462t)) and increasing at a growth rate of d(weight)/d(t) = 0.462y exp (0.221 - 0.462t) with the maximum achievable weight increment per month as 1.517kg.

The nonlinear growth pattern overtime was further confirmed and the trend analysis developed in Gompertz and logistic growth models. The 3-parameter Gompertz and logistic growth models were statistically significant at the 5% significance level. From the model selection criteria, Gompertz model with the lower values of root mean square error and residual deviance compared to the logistic model, was selected as better growth model as the growth pattern and for reliable prediction of weights of babies in the municipality at 97.4% variability.

Finally, all the weights of babies that we considered in our study established that growth of weights of babies studied is not linear but indicated non-linear growth pattern of Gompertz and/or logistic over time.

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