Competent and uncomplicated PID control algorithm design expressions for controlling second order systems

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Abstract: Proportional-integral-derivative (PID) control algorithm with its various forms, namely P (proportional), PI (proportional-integral), PD (proportional-derivative), PID, and compensators, is considered the most applied and widely used control algorithms in industry; this is because of its simple construction, robustness and capabilities to achieve desired control over plant performance. Designing PID algorithm is accomplished with a compromise to result in an overall system responds with acceptable levels of stability, response fastness, smoothness and costs. Various PID design methodologies and expressions have been introduced in text and literature,each has its advantages, disadvantages and limitations. In the present work, a new, efficient, simple, easy to apply and linear expressions for PID algorithmP-. PI, PD, and PID modes design are derived and presented. Expressions are derived based on relating parameters of both controller and plant. The expressions are intended to control the behavior of second order systems and approximated as such systems, such that it responds with acceptable stability level, minimum possible overshoot, oscillations and steady state error. To further improve the resulted response, only one tuning parameter ∝, is introduced

To test, analyze, and evaluate the design expressions, MATLAB/Simulink software was used. A simulation model was built by integrating the next sub-models; PID control algorithm modes, drive with limitation and saturation blocks, sensor, desired output signal generator and finally various forms of second order systems. The simulation model was developed and refined to be as close as possible to real life conditions and processing. Studying the recorded testing, graphical and numerical results show that the suggested expressions are being simple and easy to apply, and also efficient in providing better control over controlled system's behavior in terms of response measures and performance indices.Moreover, expressions are efficient to speed up response and eliminate or reduce overshoot, rise time and settling time. Studying results show that, the overall system responds with acceptable fast response, in most cases without overshoot, oscillation and with minimum steady state error and better ISE and IAE indices values.

Keywords: Optimum control system; PID modesdesign; approximation; second order system.

1. Introduction

With modern advances in systems design process and its production technologies, one of the most, not only important but also influential decisions, in modern systems design process, is the one related to choosing, integration and design of two components, that are directly related to each another, namely; controller/physical control unit and control program/algorithm. The final overall system's performance, quality, reproductively and efficiency depends highly on these two components. Many factors affect the decision about these two components mainly; system complexity desired system performance, functionality, efficiency, precision, costs.

Control systems play very important role in modern systems design and construction; they are one of the main parts in many areas, from industry to human's everyday life, to Nuclear power plants. It is used to enrich performance, quality, reproductively, efficiency, adaptability and safety. Control system is a term used to describe a system that consists of many components, namely; Sensor, Controller, physical control unit, control algorithm/program, interfaces, drive, power supply, and actuator or system to be controlled. Control system design is a term that describes the control program selection, coding and design. The algorithm is the part that will command, direct, or regulate the behavior the controlled system to provide and maintain the desired response. the algorithm design process can be one the following processes; *a)* the process of selecting the control algorithm parameters-gains, *b)* for intelligent algorithm, building a knowledge rule base and Inference mechanism (engine), *c)* writing a program for e.g. PL, CNC or Microcontroller to control a given physical system [1, 2].

Thecontroller, physical control unit, arecategorized around the next main programmable forms; Microcontroller *Mc*, Personal computerPC, Micro-computer, Digital signal processors (DSP), Application specific integrated circuits (ASICs), and PLC.Meanwhile, control algorithms/program, can be built around the next main forms;event ortime driven ON-OFF, multistep and continuous algorithms. Continuousalgorithms are classified into different other forms, examples include proportional-integral-derivative (PID) in different modes (P, PI, PD, PID, and lead and lag compensators modes), feed-forward, adaptive, and intelligent algorithms.The last is further categorized into the followingform; Fuzzy logic, Neural network, Genetic and Expert Systems.

The scope of the current work is limited to suggesting simple and easy to use, linear expressions for designing continuous PID control algorithm modes to control the behavior of second order systems and systems that can be approximated as such. The design method is intended to simplify and speed up the PID control algorithm design process, and help designer, in easy and simple way, to get the system under control, and respond with acceptable system stability level, favorable fast smooth response, with minimum possible or without overshoot, oscillations and error. PID algorithm and its P-,PI-, PD- lead- and lag- modes are treated as the most commonly and greatly utilized in industry. This is because of its simple construction and robustness, as well as, its capabilities to provide the desired response in excellent way, despite systems' varied dynamic characteristics.

In literature and texts, various suggested PID algorithm design methods to achieve design goals, can be found in [3][4][5][6][7] [8] [9][10]. Each method has its application fields, limitations, advantages, and disadvantages. Most of these methods are, mainly, are developed consisting of the next repeated steps; analyzing controlled system dynamics, Control algorithm design (gains selection), simulation, testing, analysis and tuning. The most worldwide famous and applied design methods are Ziegler and Nichols method [1][11][12] Cohen and Coon [13], and Chein-Hrones-Reswick[14]. Some of other suggested methods can be summarized as follows:

Forfirst-order-plus-dead-time (FOPDT) systems, authors in [15] presented an optimal new method for PID controllers tuning, the method was developed based on utilizing twomain techniques, dimensional analysis and numerical optimization. In [16], authors designed a MATLAB built-in function is suggested for improved PID tuning, the method was suggested based on well-known Ziegler-Nichols tuning method. In [17] Simulink/MATLAB was applied as a useful tool to suggest a new method for tuning PID controllers, applying ITAE criterion to calculate controller parameters. In [18] author presented a simple and efficient expressions for model-based PID design method that was suggested based on relating parameters of both controller and plant.
Thiscurrentwork is subdivided into the following sections;in section 2: are presented the design and testing methodologies, in section 3: are derived Mathematical models of PID control algorithms modes. In section 4: expressions for designing P-, PI-, PD-, PID algorithms are presented. Insection 5: testing, suggested expression in MATLAB/Simulink, analysis, and discussion of results. Insection 6: are presented the experimental setup for System Identification process;finally Conclusions and recommendations.

2. Methodology for PID algorithm modes design and testing

The expressions for designing PID control algorithm P, PI, PD, and PID modes, as represented by Eq.(1), are derived based on relating the parameters of PID control algorithm, namely; K_P, K_I, K_D, T_I, T_D gains, to parameters of second order plant to be controlled, namely; time constant T, damping ratio ξ , system dc gain KDC, undammed natural frequency ω_n and desired output reference value R.To derive linear, simple and easy to use PID algorithm modes design expressions, that will simplify the control algorithm design process, Eq.(1) is reduced to the smallest number of main needed system variables by applying each of the following; Dimensional analysis, mathematical modeling, system simulation, testing, evaluation and analysis of resulted step response, finally trial and error approach

To test and evaluate derived expressions in controlling the behavior of various second order systems types, MATLAB/Simulink environment is used. The software simulation model depicted in Figure 1(a), was developed assuming Microcontroller based control unit is used as physical control unit. In the simulation model, various mathematical forms of PID algorithm was used and developed as shown in sub model shown in Figure 1(b). To refine the software model and bring it as close as possible to real life conditions and values, limitation and saturation Simulink blocks were used to limit the control signal within ±5VDC range, sensor output signal within the [0-5VDC] range.Moreover, a drive circuit model was applied to amplify ±5VDC control signal value, to the power level needed to power the plant and achieve desired system response. To further evaluate derived expressionsefficiency, the resulted system response applying expressions were compared to the resulted responses, when three worldwide design methods were applied, namely; Simulink auto-tuned PID block, and two world-wide famous PID algorithm design expressions were applied namely, Chein-Hrones-Reswick and Ziegler Nichols.

To test the derived expressions generality and applicability, two sets of most common second order systems given by Eq.(2) and Eq.(3) were selected and used in the simulation model, namely, systems with small, medium and large values of time constant T, damping ratio ξ, system dc gain K_{DC}, undamped natural frequency ω_n.

To simplify and speedup analysis and evaluation process, the simulation model is built to return maximum possible data in numerical and graphical forms. The next response measures are outputted, T_R, T_S, E_{SS} and PO%. Furthermore, to further evaluate results, the given by Eq.(3), two performance indices are used, namely, IAE and ISE given. Figure 1(c) show the Simulink sub-model for reading error, IAE and ISE performance indices

$$K_x = f_x\,(T, K_{DC}, R, \omega_n, \xi) \qquad (1)$$

$$G_{sys(1)}(s) = \frac{4}{s^2+1.5s+1}$$
$$G_{sys(2)}(s) = \frac{3}{2s^2+5s+1} \qquad (2)$$
$$G_{sys(3)}(s) = \frac{0.3}{s^2+0.3s+0.1}$$

$$G_{sys(4)}(s) = \frac{1}{10s^2+10+401}$$

$$G_{sys(A)}(s) = \frac{24.5424}{s^2+4+24.542}$$
$$G_{sys(B)}(s) = \frac{0.04}{s^2+0.2s+0.04} \qquad (3)$$
$$G_{sys(C)}(s) = \frac{17}{s^2+3s\text{^}2+19s+17}$$

$$\int_0^{\infty} e^2(t)dt$$
$$\int_0^{\infty} |e(s)|dt \qquad (4)$$

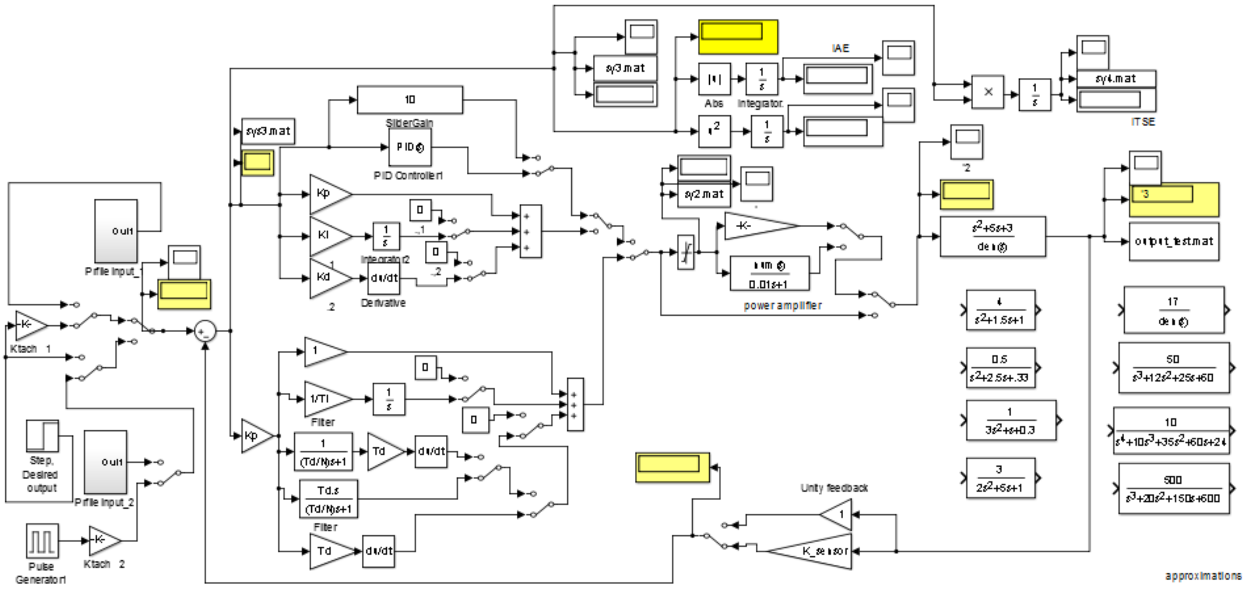

Figure 1(a) Simulink model to test the derived expressions for designing P-, PI-, PD-, PID modes control algorithms,

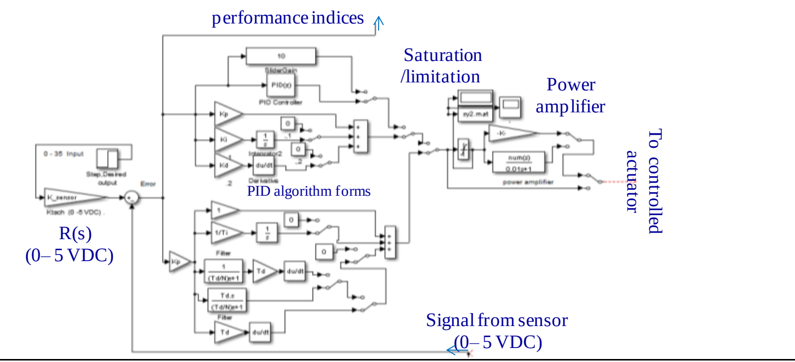

Figure 1(b) Simulinksub-modelwithapplied various PIDalgorithm forms

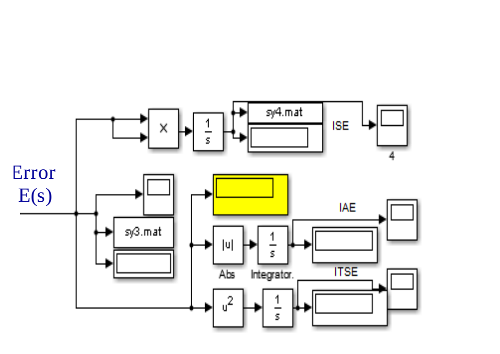

Figure 1(c) Simulinksub-model forreading error,IAE and ISE performance indices

3. Mathematical modeling of PID control algorithms

The PID control algorithms can be classified in the following modes; P-, PD-, PI- and PID forms, in addition, the lead, lag and lead-lag compensators. The proportional P-algorithm, in response (proportional) to the existing error, generates instant control signal. The derivative D-algorithm, generate control signal, only if the error changes consistently, the D-algorithm has the effect, up to a point, of giving overall system more stability, speeding up its response, reducing the magnitude of overshoot and oscillations.Finally, the integral I-algorithm integrates the error value,in the longer term, has the effect of reducing or eliminating the error value. [19].

Proportional P-Control algorithm: generates instant output control signal value *u(t)*, proportional to the existing input error value *e(t)*, as given by Eq.(5). Applying Laplace-transform and manipulating, results in P- algorithm transfer function. The P-Control signal has the effect, up to a pint, to improve overall system performance in terms of speeding up response and reducing error

The Proportional and Derivative control PD-algorithm: As shown in Eq.(6), it generates output control signal by summing two control signals; the error signal and the derivative of error signal. The PD-algorithm transfer function is given by Eq.(6) : Where: $Z_{PD} = K_P/K_D$, : PD- algorithm zero , and T_D :PD algorithm time constant.

Proportional and integral PI algorithmgenerates output signalby summing of two signals; the error signal and the integral of error signal. PI algorithm transfer function is given by Eq.(7), Where: $Z_{PD} = K_i / K_P$, : PI- algorithm zero. T_i: integral time constant

Proportional and Integral and Derivative PID algorithm generates output control signal that is proportional to three signals; the error signal and the integral of error signal and the derivative of the error signal. As shown by Eq.(8), its output signal is obtained by summing the outputs of these three algorithms. PIDalgorithm has the effect of the three P-, PD-, PI modes combined.The transfer function of the PID control algorithm can be expressed differently; e.g. in terms of integral and derivative time constants as given by Eq.(8) where: $T_D = K_D / K_P$: derivative time constant . $T_I = K_P / K_I$: integral time constant ,

As seen from Eq.(9), the PID transfer function is not causal and cannot physically be realized. To be realizable, the equation is modified by addition a lag to the derivative part, to result in the form given by Eq.(10), or by Eq.(11), where: T_D/N is the lag time constant added

$$u(t) = K_P * e(t) \rightarrow U(s) = K_P * E(s)$$
$$G(s) = \frac{U(s)}{E(s)} = K_P \qquad (5)$$

$$u(t) = K_P * e(t) + K_D \frac{de(t)}{dt} \rightarrow U(s) = K_P * E(s) + K_D sE(s)$$
$$G_{PD}(s) = K_P + K_D s = K_D\left(s + \frac{K_P}{K_D}\right) = K_D(s + Z_{PD}) \qquad (6)$$
$$G_{PD}(s) = K_P\left(1 + \frac{K_D}{K_P}s\right) = K_P(1 + T_D s)$$

$$u(t) = K_P * e(t) + K_I * \int de(t) \rightarrow U(s) = K_P * E(s) + K_I E(s)\frac{1}{s}$$
$$G_{PI}(s) = K_P + K_I\frac{1}{s} = \frac{K_I\left(s + \frac{K_I}{K_P}\right)}{s} = \frac{K_P(s + Z_{PI})}{s} \qquad (7)$$
$$G_{PI}(s) = K_P(1 + \frac{1}{T_I s})$$

$$u(t) = K_P * e(t) + K_D\frac{de(t)}{dt} + K_I * \int de(t) \rightarrow U(s) = K_P * E(s) + K_I E(s)\frac{1}{s} + K_D sE(s)$$
$$G_{PID}(s) = K_P + K_D s + K_I\frac{1}{s} = \frac{K_D\left(s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right)}{s} = \frac{K_D(s + Z_{PI})(s + Z_{PD})}{s} \qquad (8)$$
$$G(s) = K_P + K_D s + K_I\frac{1}{s} = \frac{K_D(s^2 + 2\varepsilon\omega_n + \omega_n^2)}{s}$$

$$G_{PID}(s) = K_P + K_D s + K_I\frac{1}{s} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \qquad (9)$$
$$G_{PID}(s) = K_P\frac{T_I T_D s^2 + T_I s + 1}{T_I s}$$

$$G_{PID}(s) = K_P\left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \frac{T_D s}{N}}\right) \qquad (10)$$
$$G_{PID}(s) = K_P + K_D s + K_I\frac{1}{s} = K_P + \frac{K_D s}{1 + sT_I} + K_I\frac{1}{s} \qquad (11)$$
$$G_{PID}(s) = \frac{(K_P T_I + K_D)s^2 + (K_P + K_D T_I)s + K_I}{s(T_I s + 1)}$$

4. Expressions for designing P-, PI-, PD-, PID algorithms

to simplify the control algorithm design process and help designer in easy and simple way to get the system under control, simple and linear expressions for PID algorithm modes design are presented in Table 1. The expression are suggested to design PID algorithm by calculating algorithm's gains namely, K_P, K_I, K_D, T_I, T_D. Based on controlled system parameters namely, ξ, K_{DC} ,ω_n and R. In these expressions only one tuning parameter called alpha α, is introduced, this tuning parameter is used to further improve the resulted response in terms of speeding up response an reducing overshoot, oscillation and error.

Beside appliedapproaches that are mentioned in methodology section, let us consider one extra approach based on mathematical manipulationsand assumptions. Second order systems' overall closed loop transfer function T(s), is given by Eq.(12) and represented using Block diagrams as shown in Figure 2(a)(b). Considering the next assumption; T(s) is with unity feedback H(s)=1, the forward loop system G(s),represents three components namely; the controlled system, the controller and drive , is very large, then the overall closed loop transfer function T(s), approaches unity as given by Eq. (13). Substituting in both, the PID transfer function and the general form of second order system written in terms of ξ and ω_n and further mathematical manipulation gives Eq. (14). This equation, can be used to derive the initial expressions for calculating the PID gains K_P, K_I, K_D, T_I, T_D by relating parameters of both controller's to plant's. In addition to this, and further applying this approach, the PID algorithm can be written as a second order system. Representing it in terms second order system characteristic parameters , namely the undamped natural frequency and damping ratio, gives it a new form as per Eq. (15) and Eq. (16)Where: $\omega_n^2 = K_I/K_D$ and $2\omega\varepsilon = K_p/K_D$. These equation forms can, also, be utilized to derive the initial expressions for calculating the PID gains

$$T(s) = \frac{Output}{Input} = \frac{G(S)}{1+G(s)H(s)} \quad (12)$$

$$T(s) = \frac{PID*G(S)}{1+PID*G(s)H(s)} \rightarrow \frac{PID*G(S)}{PID*G(s)} = \frac{1}{1} \quad (13)$$
$$PID * G(S) = 1 \rightarrow PID = \frac{1}{G(S)}$$

$$\frac{K_D\left[s^2+\frac{K_P}{K_D}s+\frac{K_I}{K_D}\right]}{s} = \frac{1}{\frac{\omega^2}{s^2+\xi\omega_n s+\omega^2}} \quad (14)$$
$$\frac{K_D\left[s^2+\frac{K_P}{K_D}s+\frac{K_I}{K_D}\right]}{s} = \frac{s^2+\xi\omega_n s+\omega^2}{\omega^2}$$

$$G_{PID}(s) = \frac{K_D\left[s^2+\frac{K_P}{K_D}s+\frac{K_I}{K_D}\right]}{s} = \frac{K_D\left[s^2+2\xi\omega_n s+\omega_n^2\right]}{s} \quad (15)$$
$$G(s) = K_P + K_D s + K_I\frac{1}{s} =$$
$$G(s) = \frac{(K_P T_I + K_D)s^2 + (K_P + K_D T_I)s + K_I}{s(T_I s+1)} = \frac{K_D(s^2+2\varepsilon\omega_n+\omega_n^2)}{s} \quad (16)$$

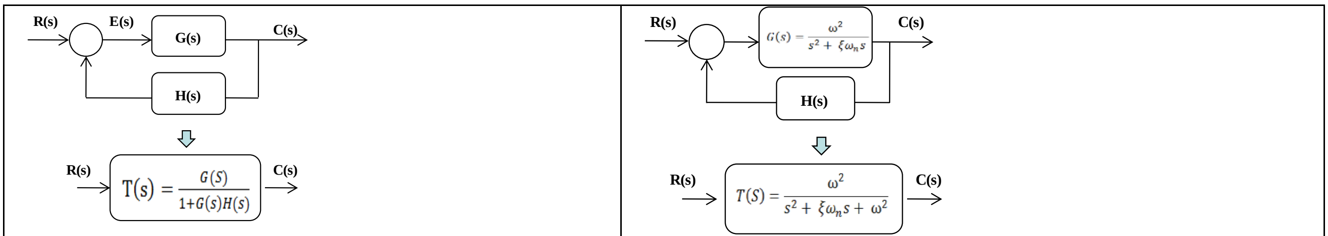

Figure 2(a) | Figure 2(b)

Figure 2:Block diagram representation: (a) closed loop system(b) closed loop transfer function second order system

Table 1: expression for PID-, PI- PD- algorithms design

Algorithm	K_P	K_D	K_I
PID	K_{DC}	$\frac{K_P}{2\,\zeta\omega_n}$	$\frac{\omega_n K_P}{2\,\zeta}$

PID For systems with big Time constant	$2\,\zeta\omega_n\alpha$	$0.2\,K_{DC}$	$\frac{0.9}{K_{DC}}$
PI	$\frac{K_{DC}}{\alpha}$	0	$\frac{\omega_n K_P}{2\,\zeta}$
PD	$\frac{K_{DC}}{0.12\,\alpha}$	$\omega_n\zeta K_{DC}$	0
PD For systems with big Time constant	$\frac{K_{DC}}{0.012\,\alpha}$	$\frac{1}{\zeta\omega_n}$	0

5. Testing, analysis, evaluation and discussion

The designed Simulink model shown in Figure 1 was used to test the applicability and correctness of the suggested expressions, evaluated and modify the derived expressions for PI-, PD-, PID algorithms design. To test the expressions, differenttypes of second order systems were applied, namely,; system that can be approximated as second order, higher order systems and real life system .

The experimental setup isdeveloped as follows;

(a) Firstly, For calculating the parameters of the controlled second order system, the system must be manipulated to result in coefficient of the s^2 term equals to unity as given by Eq.(17), then compare equation to general form of second order system given by Eq.(18). Finally, as by Eq.(19), calculate system's parameters ξ, K_{DC}, ω_n , in addition, select the desired output value reference value R.
*(b)*Secondly, the PID algorithm mode is designed using suggested design expressions, and applied to control each of systems of the two sets of second order systems and given by Eq.(2) and Eq.(3)
(c) the resulted experimental numerical and graphical data are screened, analyzed and evaluated in terms of three factors ; calculated algorithm gains, namely; K_P, K_I, K_D T_I, T_D , response measures, namely; T_R, T_S, E_{SS} ,PO% and performance indices, namely; IAE and ISE.
*(d)*The PID algorithm is designed using Simulink PID auto-tuned blockand applied to control the system
*(e)*Record resultsapplying *(c)*
*(f)*For some system, in addition to Simulink auto-tuned PID block, two worldwide famous PID algorithm design expressions are applied namely, Chein-Hrones-Reswick and Ziegler Nichols.

$$G(s) = \frac{d}{s^2 + as + b} \quad (17)$$

$$G(s) = \frac{c}{s^2 + \xi\omega_n s + \omega^2} \quad (18)$$

$$b = \omega_n^2 \rightarrow \omega = \sqrt{b}$$
$$a = 2\xi\omega_n\, s \rightarrow \xi = \frac{a}{2\omega_n} \quad (19)$$
$$K_{DC} = \frac{d}{b}$$

5.1 Testing expressions for PID algorithm design

To test the suggested expressionsfor PID algorithm design.Expressions were applied to control the behavior of each system of the two systems sets given by Eq.(2) and Eq.(3). The numerical testing results are listed in table 2 and table 3 respectively. The resulted graphical response curves are shown in Figure 3 (a-d) and Figure 4(a-c).

Table 2: testing result for PID algorithm design andresulted system behavior analysis for systems set given by Eq.(2)

sys	Method	ξ	ω_n	K_{DC}	R	α	K_P	K_I	K_D	T_I	T_D	N	5T	PO%	T_R	T_S	E_{ss}	ISE	IAE
Sys(1)	Suggested expressions	0.75	1	4	20	1	4	2.6667	2.6667	1.5	0.6667	1	6.8	0.225	0.8	6.5	0	5.179	12.15
						22	0.1818	0.1212	0.1212	1.5	0.6667	1	6.8	-	5.3	-	0	26.79	10.31
	PID tuned block					-	0.2782	0.0949	0.0888	2.932	0.3195	1.472	26.3	-	6.5	-	0.17	10.5	22.74
Sys(2)	Suggested expressions	2.166	0.5771	1.5	20	1	1.5	0.1998	0.6	7.507	0.4	1	25	-	3.2	-	0.01	8.328	24.28
	PID tuned block					-	0.61144	0.1126	0.531	5.4302	0.8684	0.950	28.2	0.01	6.5	27.6	0.01	13.28	31.32
Sys(3)	Suggested expressions	0.4743	0.3162	0.3	20	1	0.30	0.1	1.0001	2.9997	3.3336	1	43.5	0.1	24.7	23	0	20.99	56.74
						2.2	0.1304	0.0435	0.4348	2.9997	3.3336		39.3	-	20	-	0	38.31	101.3
	PID tuned block					-	0.2719	0.0438	0.2694	6.2078	0.9908	1.8614	55	-	20.9	-	0.14	35.24	77.57
Sys(4)	Suggested expressions	1.7678	0.7071	1.5	20	1	0.5	0.1	0.2	5.0001	0.4	1	78.5	-	26	-	0.03	49.9	103.8
						0.2	5	1	2	5.0001	0.4	1	15	-	5.2	-	0	12.5	36.25
	PID tuned block					-	2.2397	0.638562	0.4126658	3.5075	0.1842	1.31247	15	-	5.3	-	0	12.5	36.25

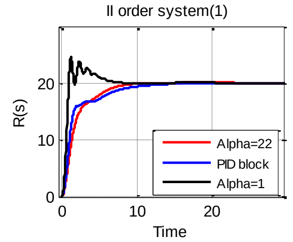

Figure 3 (a) response of system (1) : G(s)= 4/(s^2+1.5s+1)

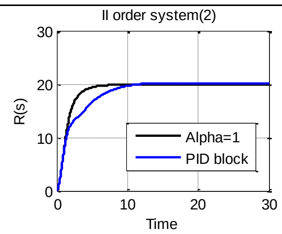

Figure 3 (b) response of system (3) : G(s)= 3/(2s^2+5s+1)

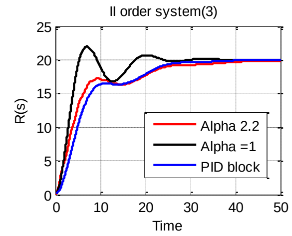

Figure 3 (c) response of system (3) : G(s)= 0.3/(s^2+0.3s+0.1)

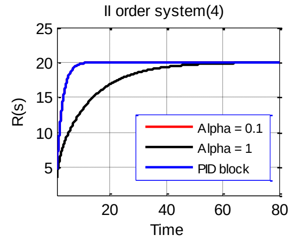

Figure 3 (d) system (4) response: G(s)= 1/(10s^2+10+40). Two responses are overlapped

Table 3: Testing result of PID algorithm design for second systems set given by Eq. (3)

SYS	METHOD	ξ	ω_n	K_{DC}	R	α	K_P	K_I	K_D	T_I	T_D	N	5T	PO%	T_R	T_S	E_{ss}	ISE	IAE
Sys(A)	Suggested expressions	0.4037	4.954	1	20	1	1	6.1357	0.25	0.1630	0.2500	1	3	0.225	0.44	2.3	0	1.828	4.993
						4	0.25	1.5339	0.0625	0.163	0.25	1	5	-	1.6	-	0	3.26	8.431
	PID tuned block						0.6879	0.4016	1.7129	0.4925	0.3640	0.6081	17.3	-	5.4	-	0.03	8.717	`14.29
Sys(B)	Suggested expressions	0.2	0.05	1	20	1	1	2	50	0.5	50	1	500	2.05	7.5	460	0	896.9	453.2
						350	0.0033	0.0067	0.1667	0.5	50.0	1	800	-	650	-	0.05	748.1	1877
	PID tuned block					Difficult to design PID algorithm													
Sys(C)	Suggested expressions	0.2425	4.1231	1	20	1	1	8.5012	0.5001	0.1176	0.5001	1	8.5	-	2.4	-	0	5.588	16.78
						0.1	10	85.0125	5.0007	0.1176	0.5001	1	7.2	-	2.4	-	0	5.588	16.78
	PID tuned block						0.627827	0.4367	0.2019	1.4377	0.3216	0.4201	16.3	-	3.55	-	0-	9.087	27.06

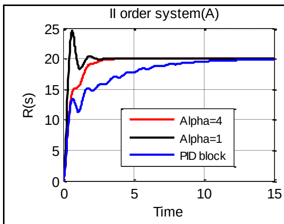

Figure 4 (a) response of system (A) : G(s)=24.542/(s^2+4+24.542)

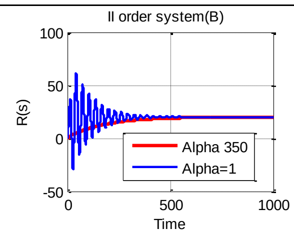

Figure 4 (b) response of system (B) : G(s)= 0.04/(s^2+0.2s+0.04)

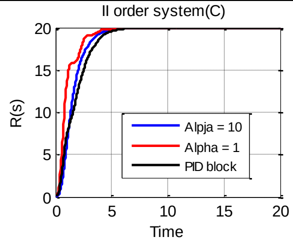

Figure 4 (c) response of system (C) : G(s)= 17/S^3+3s^2+19s+17

5.2 Testing expressions for PIalgorithm design

The numerical testing results for controlling the behavior of each of three systems given by Eq.(2) are listed in table 4. Meanwhile, the graphical testing results in terms of resulted response curves are shown in Figure 5(a-d)

Table 4: testing result of PI algorithmdesign for systems given by Eq. (2)

SYS	METHOD	ξ	ω_n	K_{DC}	R	α	K_P	K_I	T_I	5T	PO%	T_R	T_S	E_{SS}	ISE	IAE
Sys(1)	Suggested expressions	0.75	1	4	20	1	4	2.6667	1.5	15.5	0.63	0.5	16	0	6.7	14.03
						70	0.0571	0.0381	1.5	30	-	7.7	-	0	16.41	44.05
	PID tuned block					-	0.00986	0.01974	0.4995	30	-	11	-	0	29.16	95.96
Sys(2)	Suggested expressions	2.166	0.5771	1.5	20	1	1.5	0.1998	7.5065	30	-	2.4	-	0	8.335	23.29
						0.78	1.9231	0.2562	7.5065	8.5	0.0275	2.15	7.8	0	6.801	22.88
	PID tuned block					-	0.27970	0.04684	5.9708	38	-	15	-	0	33.05	81.82
Sys(3)	Suggested expressions	0.4743	0.3162	0.3	20	1	0.3	0.1	3	90	0.275	5.4	75.5	0	38.2	83.3
						3.5	0.0857	0.0286	3	73	-	27.5	-	0	58.31	156.9
	PID tuned block					-	0.01316	0.026322	0.5	70	-	18.5	-	0	60.82	205.8
Sys(4)	Suggested expressions	1.7678	0.7071	0.5	20	1	0.5	0.1	5.0001	60	-					
						0.1	5	1	5.0001	15	-	5	-	0	12.5	38.25
	PID tuned block					-	0.89431	0.22289	4.0123	40	-	10	-	0	19.92	44.79

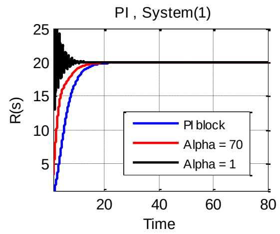

Figure 5(a) response of system (1) L G(s)=4/(s^2+1.5s+1)

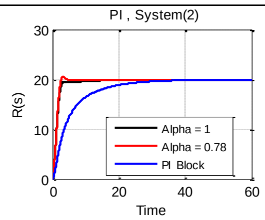

Figure 5(b) response of system (2) G(s)=3/(2s^2+5s+1)

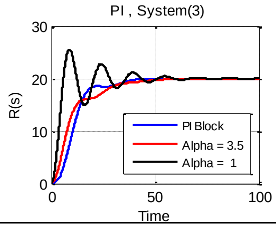

Figure 5(c) response of system (3) G(s)= 0.3/(s^2+0.3s+0.1)

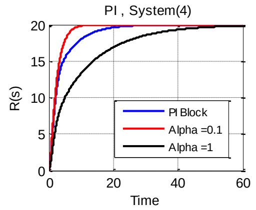

Figure 5(d) response of system (4): G(s)=1/(2s^2+5+2)

5.3 Testing expressions for PD algorithm design

The numerical testing results for controlling the behavior of each of three systems given by Eq.(2) are listed In table 5 The graphical testing results are shown in Figure 6 (a)(b)(c).

Table 5: Testing result of applying PD algorithm design expressionsfor systems given by Eq.(2)

SYS	METHOD	ξ	ω_n	K_{DC}	R	α	K_P	K_D	T_D	N	5T	PO %	T_R	T_S	E_{SS}	ISE	IAE
Sys(1)	Suggested expressions	0.75	1	4	20	1	33.3333	2.8284	0.0849	1	2.4	0.075	0.8	2.1	0.15	3.077	10.8
						1.6	20.8333	2.8284	0.1358	1	2.4	0.005	0.8	2.1	0.24	3.379	10.81
	PD tuned block					-	13.991	13.711	0.9799	6269.	6.2	-	4	-	0.35	8.291	15.84
Sys(2)	Suggested expressions	2.166	0.5771	1.5	20	1	12.5	1.875	0.15	1	9.5	0.05	7	8.7	1	23.54	59.88
						0.1	125	1.875	0.015	1		0.01	7	10.5	0.11	18.54	56.5
	PD tuned block					-	28.700	11.0563	0.385	643.5	10.5	-	7	-	0.45	20.46	58.75
Sys(4)	Suggested expressions	1.678	0.7071	3	20	1	25	3.75	0.15	1	4	-	2.1	-	0.26	6.579	22.82
						0.5	50	3.75	0.0750	1	3.5	0.005	2.1	2.8	0.13	6.538	22.79
	PD tuned block					-	25.346	7.65627		47.139	4	-	2.3	-	0.26	6.888	22.85

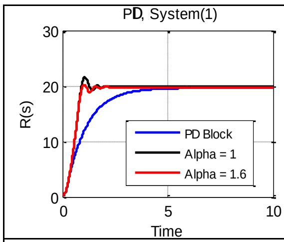

Figure 6 (a) response of system (1) :G(s)=4/(s^2+1.5s+1)

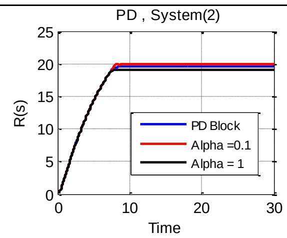

Figure 6 (b)response of system (2): G(s)=3/(2s^2+5s+1)

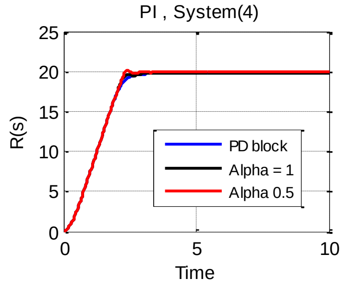

Figure 6 (c) response of system (4) , G(s)= 3/(2s^2+5s+1)

5.4 Analysis, Evaluation and Discussion of testing results

the resulted system numerical data recorded in Tables2-6 and graphical response curvesshown in figures 2,4,5 for controlling the behavior of each system was screened, analyzed and compared in terms of resulted response measures, TR, TS, ESS, PO% and performance indices IAE and ISE. analysis show that the suggested expressions arecorrect, applicable and effective for designing every mode of the PID algorithm namely; PI-, PD-, or PID-modes, in simple and easy to use way, to result in much better results in terms of stability level and response speed, than when the same algorithm is designed using auto-tuned PID block or Ziegler Nichols methods.In this contest, the expressions are effective to eliminate or reduce overshoot and error, as well as, speed up resulted system, in terms of reducing rise time and settling time.

The results data show that, the expressions are effective for designing PID algorithm modes , such that most used second systems, respond smoothly with acceptable fast response, almost without any observable overshoot or oscillations and with minimum error, in addition, lower values of ISE and IAE performance indices .in addition, results show, that the suggested only one tuning parameter alpha α , can help in improving resulted response and achieving desired response in terms of speed, less oscillation and error

The suggested expressions for PD algorithm design, show a shortage for controlling system with very big time constant e.g. T=10, T=15 seconds.

5.5 Testing design expressions for higher order systems and Discussion

PID control algorithm modes are dominating in controlling industrial applications; this is due to the fact that PID algorithms operate very well, mostly with first order system dynamics. However, in different cases and higher order system forms, the algorithm just will not perform very well, such cases include; (a) to achieve a tight and precise control over higher order systems,(b) plants with big time delay L, (c) lightly damped oscillatory modes systems.
To further test and evaluate both efficiency and applicability of suggested expressions, different higher order systems were used, namely; third and fourth order systems, system with positive pole, and systems with small and big time constants.The transfer functions with its approximated second order form, are given by Eqs(20) by (24). For these systems, the resulted response were recorded, analyzed and compared with response resulted when algorithm is designed using PID auto-tuned Simulink block. Moreover, to further evaluate the design expressions, the resulted responses were compared with responses when the algorithm gains design were done applying two famous method; Ziegler-Nichols andChien-Horens-Rewswik.

The suggested methodology for designing Algorithms based on calculating controlled system parameters,is based on approximating the higher order system to it second order approximated system, and use the approximated system to design the algorithm gains, then the designed gains are applied to control the original system to achieve desired output value of 20 with smooth, stable, and fast response.

Testing the design expressions for designing PID algorithms for controlling each of systems given by Eqs. (20) to (24). Usingeach system's approximated as second order systems form to calculate the K_P, K_I, K_D, T_I, T_D gains, resulted in graphical response curves shown in Figure 7(a-e) and recorded numerical data listed in Table 6.
Studyingthe recorded graphical and numerical data, showthe following:

(1)Designing PID algorithm gains applying suggested expressions, results in much better control over systems in terms of response measures and performance indices, than when PID gains are designed applying auto-tuned PID block.
(2) It is also reveal that expressions are efficient to speed up response and eliminate or reduce overshoot, rise time and settling time. The overall system responds with acceptable fast response, in most cases without overshoot, oscillation and with minimum steady state error and ISE and IAE indices values.
(3)Thesuggested only one tuning parameter alpha a , can help in improving resulted response and achieving desired response in terms of speed, less oscillation and error
(4)The drawbacks are in that the tested fourth order system responds with an error value of 0.08, while applying other three design methods, system respondswithout error.

$$G(s) = \frac{17}{s^3+3s^2+19s+17} \quad G(s) = \frac{17}{s^2+8.2810s+17} \quad (20)$$

$$G(s) = \frac{50}{s^3+12s^2+25s+50} \quad G(s) = \frac{5}{s^2+2s+5} \quad (21)$$

$$G(s) = \frac{10}{s^4+10s^3+35s^2+50s+24} \quad G(s) = \frac{0.833}{s^2+3s+2} \quad (22)$$

$$G(s) = \frac{500}{20s^3+20s^2+150s+500} \quad G(s) = \frac{5}{s^2+6s+5} \quad (23)$$

$$G(s) = \frac{s+1}{s^2+3s+1} \quad G(s) = \frac{1}{s^2+3s+1} \quad (24)$$

Table 6: PID algorithm design for higher order systems that can be approximated as second order systems

SYS	METHOD	ξ	ω_n	K_{DC}	R	α	K_P	K_I	K_D	T_I	T_D	N	5T	PO%	T_R	T_S	E_{ss}	ISE	IAE
Third order system	Suggested expressions	0.2425	4.1231	1	20	1	2	0.9	0.20	2.2222	0.1	1	7.5	-	2.4	-	0	5.588	16.78
						10	20	0.9	0.2	22.2222	0.01	1	7.5	-	2.4	-	0	5.588	16.78
	PID					-	0.6	0.9	0.3	0.7	0.48	2.	8	-	2	-	0	6.	22.

	tuned block						7845	1981	281	376	36	0675			.7			908	95
Third order system	**Suggested expressions**	0.4472	2.2361	1	20	1	2	0.9	0.2	2.2222	0.1	1	12	0.025	1.1	-	0	5.588	13.78
						0.1	0.2	0.9	0.20	0.2222	1	1	8	0.025	1.6	-	0	5.774	18.33
	PID tuned block						0.3894	0.56429	0.16806	0.6901	0.4316	0.6775	14	-	4.1	-	0	8.859	23.37
IV order system	**Suggested expressions**	1.0607	1.4142	0.4165	20	1	3	2.1609	0.0833	1.3883	0.0278	1	8	-	3	-	0	7.5	22.92
						25	75	2.1609	0.0833	34.7083	0.0011	1	8	-	3	-	0	7.5	22.92
	PID tuned block					-	6.9892	4.99645	2.42489	1.3988	0.3469	223.09	8	-	3	-	0.01	7.533	22.93
Third order system	**Suggested expressions**	1.3416	2.2361	1	20	1	6	0.9	0.2	6.6667	0.0333	1	6	0.1150	0.6	4.4	0	2.77	5.623
						0.01	0.06	0.9	0.2	0.0667	3.3333	1	2.2	-	0.75	-	0	2.743	8.516
	PID tuned block					-	0.7224	2.0195	0.06365			831.113	1.7	0.075	0.33	1.3	0	1.176	4.19
II order with positive pole	**Suggested expressions**	1.50	1	1	20	1	3	0.9	0.2	3.3333	0.0667	1	6.2	-	1	-	0	2.234	4.88
						10	30	0.9	0.20	33.3333	0.0067	1	4.2	-	0.6	-	0.06	1.813	4.514
	PID tuned block					-	1.1043	2.15509	0.0690	24.387	xx	xx	5.5	0.082	0.78	4.8	0	2.562	5.149

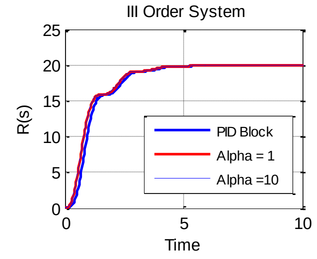

Figure 7(a) Testing third order system

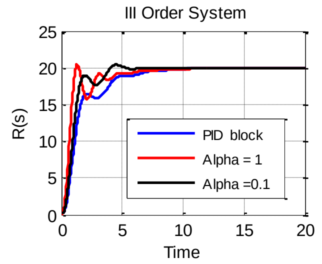

Figure 7(b) Testing third order system

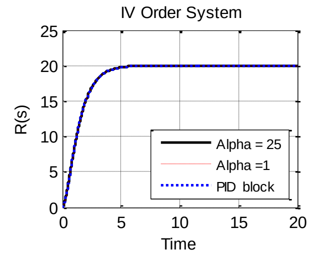

Figure 7(c) Testing fourth order system

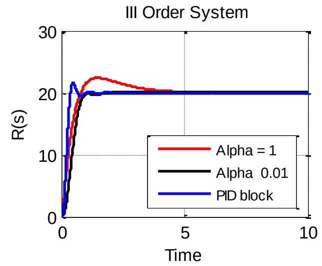

Figure 7(d) Testing third order system

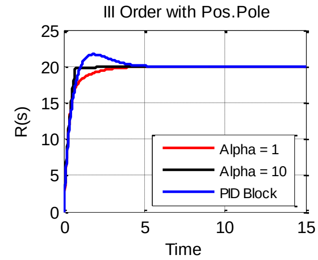

Figure 7(e) Testing third order system with positive poles G(s)=(S+1)/s^3+3s+2

5.5.1 Comparingdesign expressions with other PID algorithm design methods

To further evaluate suggested expressions, the resulted graphical and numerical results were compared with resultswhen three PID design methods are applied, namely; Simulink PID auto-tuned block, Chein-Hrones-Reswick and Ziegler Nichols. Thefourth order system and its approximated second order system form given by Eq. (25) was used for testing and comparison purposes.Graphical testingresults are shown in Figure 8, meanwhile numerical results are listed in Table 7.

Comparing resulted both response curves and numerical data, revealed that the designedPID algorithm gains applying expressions, gives much better results in terms of both response measures and performance indices, than when it is designed applying the world wide famous PID algorithm design expressions,as well as, auto-tuned PID Simulinkblock.

$$G(s) = \frac{10000}{s^4+126s^3+2725s^2+12600s+10000} \quad (25)$$
$$G(s) = \frac{5}{s^2+6s+5}$$

Table 7: PID algorithm design for higher order systems applying different algorithm design methods, namelysuggested expressions, Chien-Horens-Rewswik Ziegler-Nichols, and PID auto-tuned Simulink block.

SYS	METHOD	ξ	ω_n	K_{DC}	R	α	K_P	K_I	K_D	T_I	T_D	N	5T	PO%	T_R	T_S	E_{ss}	ISE	IAE
	Suggested					1	6	0.9	0.2	6.666	0.0333	1	4	0.0	0.	2.	0	2.6	9.077

IV order system	**expressions**	1.3416	2.2361	1	20					7				95	77	4		01	
						0.35	2.1	0.9	0.2	2.3333	0.0952	1	2	0.02	0.76	2.S	0	2.496	9.047
	Ziegler-Nichols					-	11.1524	34.3786	0.9045	0.3244	0.0811	1	3.4	1.7	0.74	2.7	0	4.799	15.02
	Chein-Hrones-Reswick					-	5.5762	0.8632	0.1231	6.4599	0.0221	1	6	0.3465	0.74	5.4	0	4.328	10.83
	PID tuned block					-	0.59025	0.48965	0.14023	1.2055	0.2376	130.54	5.5	0.025	1.75	5.2	0	4.566	13.86

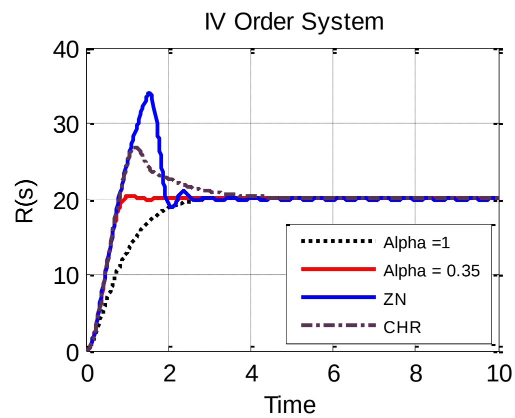

Figure 8 algorithm design for higher order systems applying different design methods

6. Experimentalsetupfor system identification

Designing PID algorithm modes is accomplished by calculating controller parameters, namely K_P, K_I, K_D, T_I, T_D gains, The calculations are done using the controlled system parameters; namely damping ratio ξ, dc gain K_{DC} , undamped natural frequency ω_n and desired output reference value R.

For real life system, to obtain the numerical values of these parameters, systemidentification process can be applied. As shown in Figure 9(a), to derive system's transfer function, a sensor corresponding to the variable being controlled e.g. pressure temperature, or level, is selected and interfaced to microcontroller based control unit (e.g. PIC or Arduino board). The control unit is programmed with software control program to process, save and display sensor's readings. The control unit can be further interfaced to personal computer for more data processing and analysis. Theidentification process is initiated by subjecting the controlled system to full step 100% power value input (e.g. of 220V/60Hz VAC voltage or 24 VDC).Continuously, sensor will read the changes in controlled variable values, send readings to microcontroller, that will process data and display results in graphical and numerical forms. The graphical form of the displayed data is the controlled system's open loop step response that can be used to derive the system transfer function and system parameterssystem parameters can be calculated from resulted system response as referring toFigure 9(b), and by expressions as given by Eq.(26). and Eq.(27).

$$\xi = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + ln^2(\%OS/100)}}$$

$$\%OS = \frac{M_P}{Y_{final}} * 100\% \qquad (26)$$

$$T = \frac{1}{\omega_n \xi}$$

$$5T = 99.9 * y(t)$$

$$T_s = \frac{4}{\omega_n \xi} \qquad (27)$$

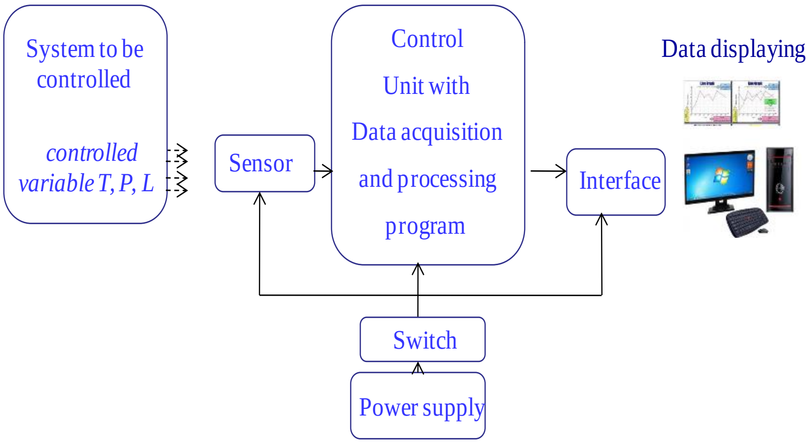

Figure 9(a): The identification processrepresented using block diagram.

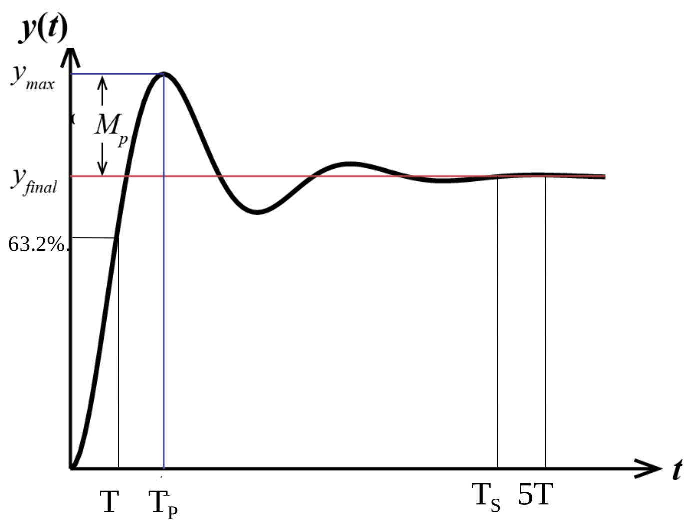

Figure 9 (b): General form of second order undamped system response.

Conclusions

Based on studying, analysis and evaluation of resulted graphical and numerical data of applying suggested expressions for PID algorithm modes design, the following is concluded

(a) For controlling the behavior of second order systems, and higher order systemsapproximated as such systems, simple and easy to uselinear expressions for designing PID control algorithm, PI, PD, and PID modes, are derived and tested.

(b) the expressions are with only one tuning parameter called alpha α, that is used to further improve the resulted response in terms of speeding up response an reducing overshoot, oscillation and error
(c) Expressions simplify PID control algorithm modes design process, and got systems under control and to respond with acceptable stability level, response fastness, without or with minimum possible overshoot, oscillation and steady state error.
(d) analysis, studying andComparing recorded both graphical response curves and numerical data when other PID algorithm design methods are applied, show that the suggested expressions beside being simple and easy to apply, are also efficient in providing better control over system's behavior in terms of response measures and performance indices, moreover, Expressions are efficient to speed up response and eliminate or reduce overshoot, rise time and settling time. The overall system responds with acceptable fast response, in most cases without overshoot, oscillation and with minimum steady state error and ISE and IAE indices values.
(e) The suggested only one tuning parameter alpha a , can help in improving resulted response and achieving desired response in terms of speed, less oscillation and error
(f) The drawbacks are in that the fourth order system responds with an error value of 0.08, while other three methods respond without error.

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