Development of an Iterative Modular Expression to Accurately Adjust the Moment Capacity of a Reinforced Concrete Member

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Abstract: Every reinforced concrete member has a given limit of design requirement in two folds: a lower limit and an upper limit. A designed member would necessarily be suitable if at least the design parameter in question falls within these values. Countries, manufacturers, and regulating agencies have over the years produced and regularized codes, standards, etc. to regulate the safe adaptation of reasonable factors and advice for designers to follow.As a key component in the construction of a reinforced concrete member, reinforcing bars serve cardinal roles in effectively sustaining the element from abrupt collapse. All of this is made possible by adequately addressing the requisite amount of reinforcement that can withstand said moment or torsion induced on the member. However, the placement and location of a rebar may have the inability to offerstructural importance to the overall performance of the structure. Hence, the iteration procedure developed herein has the capability to compare in real time the changing effect of one parameter against the rest of the other parameters in the equation under consideration. Key components considered in the steel ratio equation include the design moment capacity, the section effective area, the compressive strength of the concrete, etc. hence these are parameters that go constantly into the iteration. An arbitrary adjustingfactor is selected by the design engineer. This factor is the key component controlling the entire iteration procedure. It smoothens the linear and parabolic relationships that exist between any two parameters and forthose that exist amongst several parameters. In achieving this goal, several mathematical expressions are formulated thus leading to the establishment of specific hypotheses between parametric relationships. Here, a simple span railway bridge deck is analyzed up to the determination of its area of steel and then begins the iteration to adjust all necessary parameters where needed.

Keywords: Iteration, moment capacity, steel ratio, compressive strength of concrete, yield strength of steel, flexure, load factor design, AREMA, ACI,

1. Introduction

Structural elements are designed particularly on the mode of application of the imposed load and their response to these loads. In the analysis and design of a member, there are numerous combinations of possible loading conditions that exist. These loading categories also differ from element to element based on how these loads are applied. Notwithstanding, every structural analysis has a single goal: to accurately understand the magnitude of load to be applied on the section/elementand then to provide the requisite amount of resisting elements needed to balance the incoming loads while at the same time not compromising the structural integrity of the element itself.With technological advances nowadays on an exponential increase, alternate methods of reinforcement are being rapidly studied and adopted into texts of codes and specifications for public use. Not essentially only the reinforcing bars, even the concrete strength hasincredibly improved but to a limiting value of seventy mega-pascals (70 Mpa) for design consideration.Above this value, it has been widely experimented and reported thatthere would be several inaccuracies(Committee, 2019)(AREMA, 1999).However, reinforcing/reinforcement bars, technically referred to as rebars have been the core reinforcement element in the past.(Committee, 2019)and several preceding codes had only minutely addressed reinforcement relating to fiber reinforcing elements such as fiber reinforced polymer (FRP)whereas theylengthilydiscuss steel reinforcement.Inasmuch it's not widely used comparatively to steel, it has capabilities to be used as an external reinforcing element thus even serving well in saline environment(Nour et al., 2007).This is a one key advantage elements such as these have over the traditional reinforcing steel.

As the construction industry progresses, hence more improvements emanate. Major capital projects are rarely constructed in-situ; these structures are normally precast pretensioned or post-tensioned and then fitted into position.

Due to some of the many challenges faced by most engineers to accurately design bridge structures (Yücel et al., 2020) developed a mini program to roughly calculate basic dimensional properties of railway and highways bridges in line with some of the most internationally recognized codes. Thus, having a fair idea from experience and practiceabout what the imposed loadingfor a specific member or section are, also helps inform the designer about an approximate dimension of the section.

1.1 Basic Load Combinations

In analysis and design, the basic input is the applied load. However, in different instances, the load may appear as a single form of load either concentrated, or distributed and or as a combination of several loads as also concentrated or distributed. Logically, stresses can also be considered as distributed body force in a member.The combined action of these loads leads to the adoption of the required steel area needed to withstand the incoming load. Due to the mode and magnitude of the recognized application of the load, strength reduction factors, probable combination, etc vary from code to codewhen using load-factor or load-factor-resistance design method. This can be seen in the comparable tables below.

Group	Item
I	1.4 (D + 5/3 (L + I) + CF + E + B + SF)
IA	1.8 (D + L + I + CF + E + B + SF)
II	1.4 (D + E + B + SF + W)
III	1.4 (D + L + I + CF + E + B + SF + 0.5W + WL + LF + F)
IV	1.4 (D + L + I + CF + E + B + SF + OF)
V	Group II + 1.4 (OF)
VI	Group III + 1.4 (OF)
VII	1.0 (D + E + B + EQ)
VIII	1.4 (D + L + I + E + B + SF + ICE)
IX	1.2 (D + E + B + SF + W + ICE)

Load combination	Equation	Primary load
$U = 1.4D$	(5.3.1a)	D
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1b)	L
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$	(5.3.1c)	L_r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1d)	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	(5.3.1e)	E
$U = 0.9D + 1.0W$	(5.3.1f)	W
$U = 0.9D + 1.0E$	(5.3.1g)	E

Table 1: Possible load combinations by AREMA (left) and by ACI (right)

2 Design Discussions

A key point to note between the two tables is that the American Railway Engineers and Maintenance-of-Way Association (AREMA), identifies extra loads developed during the operation of a given locomotive and inserted them into the possible combinations. These loads are Impact load, wind load on live load, and longitudinal force. The pure load (weight) exerted by the standardized locomotive or train in itself constitutes the live load in the case of the AREMA combinations.However, in the case of the American Concrete Institute (ACI), these extra loads are not necessarily added because of the modal occupancy of these structures. With these several combinations included in a railway structure, in this case, a bridge; it is the responsibility of the designer to logically select which possible combination to adopt. The designer, with reconnaissance data available, would then further deselect or remove from an equation those parameters that are structurally not feasible for inclusion in the analysis. The same also happens in the case of adopting the ACI code as well as any other code. By the way, it should be clearly noted that codes are not restrictive documents; more especially so when it's not legally adopted by a nation. They do provide advice for better practice and uniformity in the specific field of adoption. In the judgement of the design engineer, sound engineering discretions can be taken to avoid any foreseeable circumstances.

2.1 Scope

In the procedure leading to the development and formulation of the mathematical models, technical design decisions have to be made. As a simple single span structure, the following assumptions can be drawn:

a. The structure is statically determinate
b. The structure is cast-in-place
c. The environment is considered normal
d. The design live load is not appliedon the structure
e. Torsional effect is negligible, and that
f. Best practice is adopted during the construction process

An adequaterepresentation of these assumptions effectively guides the formulation process.

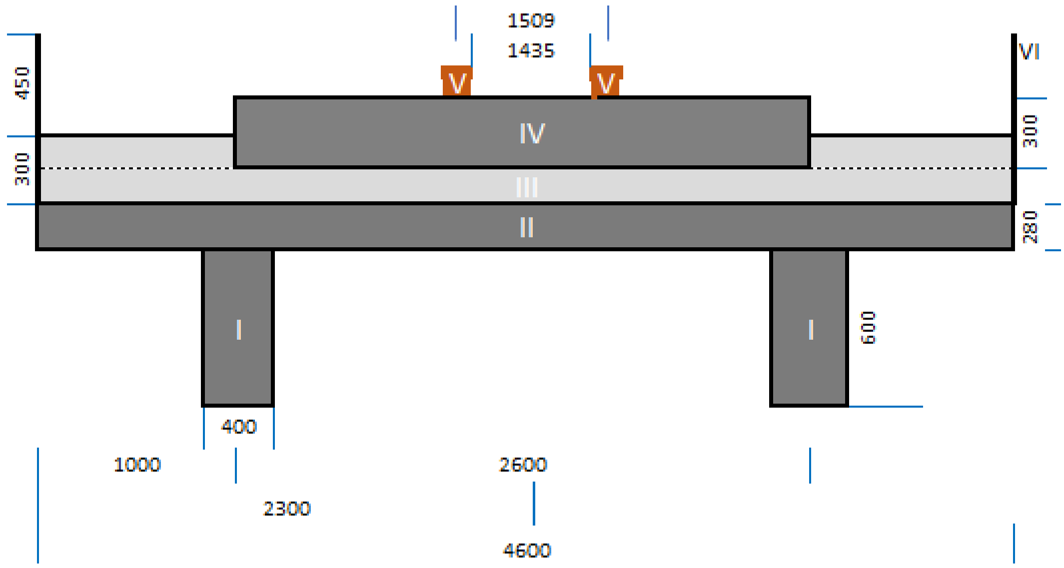


Figure 1: A simple representation of a single-track railway bridge

When a structure such as this railway bridge is statically determinate, it gives more leverage to an increased member size than its indeterminate counterpart. Theoretically, a structure designed as both a determinate and an indeterminate member can be identified from the final dimensions of its component parts.

As a cast-in-place (i.e., in-situ)structure of such minimum length required of a bridge, there would be no need for an expansion joint (Ramos et al., 2019) in the rail as well as the deck. The length plays akey role in the application of joints in the deck and rail. Longitudinal forces do develop high stresses in these continuous members and as such these joints help reduce these stresses. Allowing the full application of these loads (i.e., by ignoring the application of joints), leads to a condition wherein more reinforcement would likely have to be provided in the specific region where needed. This is a thematically considered point. Adjusting the load magnitude would as well lead to change in the reinforcement magnitude.

A structure built in a moist and wet environment will have to satisfy other extra environmental requirements that other environmental conditions may not have to. Several works including (Committee, 2019; Id, 2015)(AREMA, 1999)(McCormac, 2015) all specify maximum cracking widths for structures built in areas of extreme or moderate exposure conditions. A normally recommended solution is to increase the cover concrete. Concrete cover is normally the unreinforced concrete outside the effective depth that provides protection to the reinforcing bars.

In the next research work, the inclusion of live load in the combination would take effect. In bridge analysis, live loads have high magnitudes compared to their dead load counterparts.

Torsion causes a member to twist under applied loads.Most notably occurring in cantilever sections, torsional forces induced in a member would require extra reinforcement criteria to be satisfied like a more rigid and firmer mode of stirrups applications(McCormac, 2015). Bridges that may contain a turnout track will cause torsion as well (Siew et al., 2017), though depending on the angle of the turnout. Other loads including seismic loads also impact torsion.

The best designed structure may not utilize its full potential if the construction does not follow design accordingly. Except where field conditions do not permit, and where change order is required, design shall always be followed. (Akbari, 2019), researching the types of short-span railway bridges, listed three techniques and noted that the slide-in construction method was preferrable over the other two. However, this is in the case of a cast-in-place bridge.

2.2 Materials and section properties

Now, these are key data that can be used in the design analysis.Some are not necessarily involved into the iteration but they present a further knowledge on the properties of the material. For instance, the elastic modulus of the steel andconcrete used can give a fair idea about their response to deflection though deflection is not considered herein.

Description	Magnitude		Remark
Clear span (s)	6	m	Designer's pick due to site condition
O/c of gir. Span (s_g)	2.6	m	Designer's pick, girder length
Ties spacing (s_t)	60	cm	Designer's pick, due to track stiffness parameter
Ties length (L_t)	2.6	m	German B70 sleepers details
Ties weight (w_t)	1.3	kN/m	German B70 sleepers details + add-ons
ballast unit wt. (w_b)	19	kN/m^3	AREMA 8-2.2.3 b. (2)
Weight of rail (w_r)	3	kN/m	AREMA 8-2.2.3 b. (2)
f'_c	35	MPa	METROLINX RC-0506-04STR Part 3-11.1
f_{yk}	420	MPa	AREMA 8-2.3.2 (c., d.), 19.4.2.2.2
Conc. Elast. Mod. E_c	28	Gpa	AREMA 8-2.23.4a.
Steel Elast. Mod. E_s	200	Gpa	AREMA 8-2.23.4b.
Slab bar #	25		Designer's pick
Girder bar #	32		Designer's pick
Concrete Cover (cc)	50	mm	AREMA 2010 Chapt. 8-2.6.1 (Table 8-2-7)
concrete density	24	kN/m^3	AREMA 2.2.3 b. (2)
Ø	0.90		AREMA 2.30.2 b.
ψ	0.85		AREMA 2.30.2 b.
Live Load (LL)	360	kN	AREMA 2.2.3 c.
	120	kN/m	AREMA 2.2.3 c.

Table 2: Material and sectional properties of the 6-meter RC Railway Girder Bridge

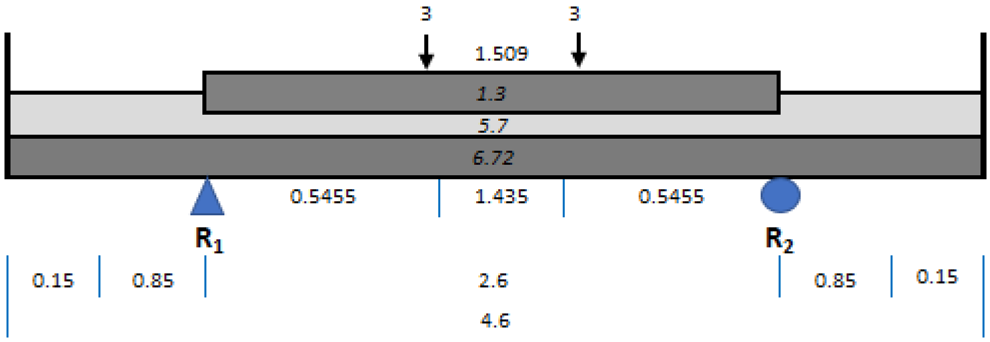
For the structure to sustain its own weight, dead load acting the structure (i.e., the structure's own weight) has to be designed for. All these loads act in the transverse except for the rail. Because structures of such are designed in a 1-meter band in the SI Unit (or 12-inch band) in the US Customary unit, loads that are distributed per unit meter in the longitudinal direction can be considered concentrated over that one-meter band. This is specifically true in the case of the rail when considering the design of a transverse member such as the deck under consideration.

A. DEAD LOAD (DL)

Desig-nation	L (m)	W (m)	Thk. (m)	Density (kN/m^3)	Unit weight (kN/m)	Point Load (kN)	Remark
II.		1.00	0.28	24	6.72		Deck
III.		1.00	0.3	19	5.7		Ballast
IV.					1.3		Tie
V.	1.00	1			3	3	Rail

Table 3: Dead Load table of a 1-meter width RC Railway Girder bridge

The loads outlined in the table are represented on the symbolic deck. To protect the ballast from fall-over, a perforated (i.e., only from above the ballast line) steel platewith negligible weight is fitted against the deck.



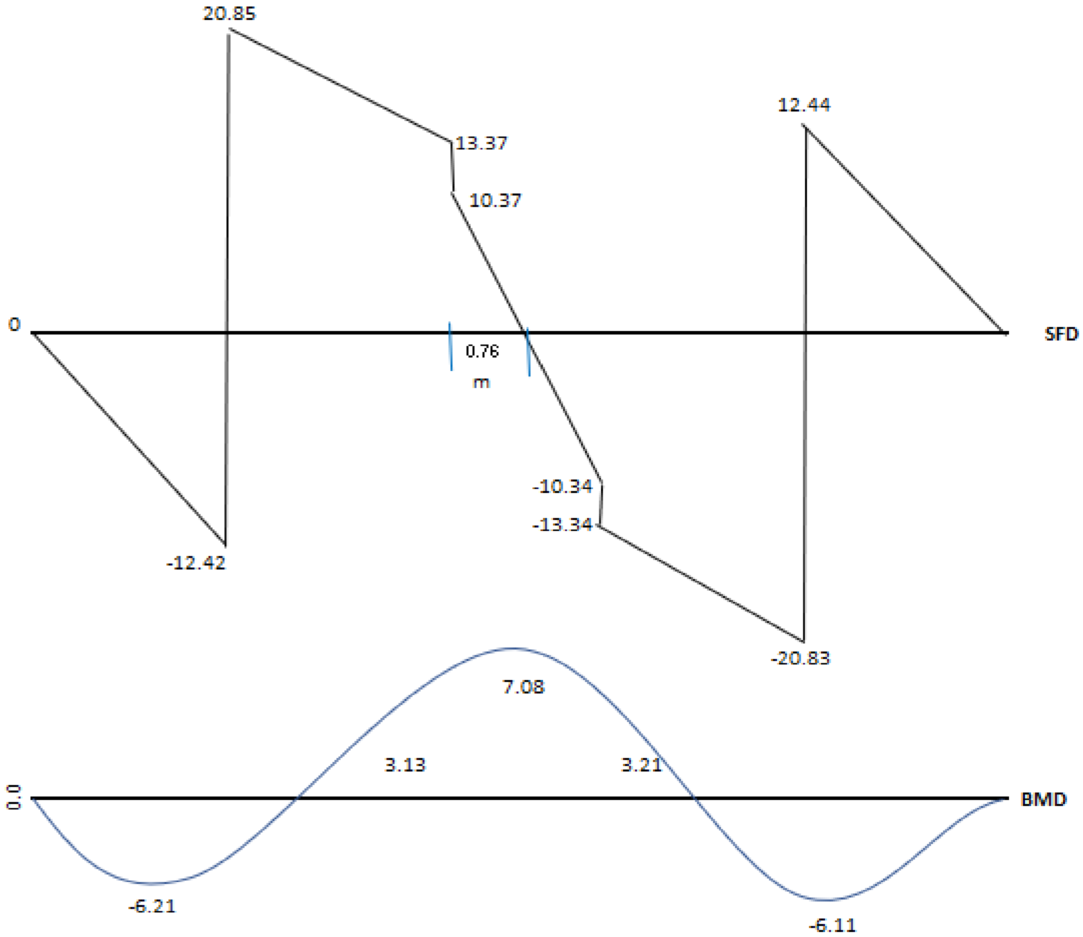


Figure2: Shear and moment diagrams representation the railway bridgedeck under consideration

The analysis provides the following summary:

- Maximum shear (V_{max}) = 20.85 kN
- Maximum moment (M_{max}) = 7.08 kN.m
- Point of zero shear occurs at mid length of deck
- The first point of contraflexure occurs 0.19 m away from the left support and
- The second point of contraflexure occurs 0.19 m away from the second rail

$$PI_1 = \frac{3.13}{0.5 \times 34.22} = 0.19 \text{ m}$$

$$PI_1 = \frac{3.21}{0.5 \times 34.17} = 0.19 \text{ m}$$

Design for Flexure [Negative Moment (at the constraints)]

The combination of loads on a structural member takes into account the most possible combination and their proper factors by which each load and or moment is multiplied.The shear or moment for the possible load combination in AREMA Group III or ACI Group I is obtained as outlined in Table 1 above.

$$\mathbf{M = 1.4 (D + L + I + CF + E + B + SF + 0.5W + WL + LF + F}$$

Adopting the load factor design method, the factored design moment for the dead load exclusively is obtained by limiting all other moments! This implies that only the dead load moment is considered.

$\mathbf{M_{u,DL} = Ø[1.4M_{DL}]}$, where Ø =0.90,

$M_{u,DL}$=8.9208 kN.m

This is the maximum design moment caused by the application of all the calculated dead loads. It also includes effects of the cantilever section. It is then visualized that the maximum moment is at the mid-span of the deck because of the negligence of the weight of the perforated steel plate as well as the placement of the sleepers/ties. These sleepers are directly end spanning at the center of both girders.

(AREMA, 1999)Table 8-2-10 recommends the depth of some primary load-carrying members. The depth of the slab can be estimated as:

$$t_{deck} = \frac{s_g + 3}{20}$$

$$t_{deck} = \frac{2.6 + 3}{20} = 0.28 \text{ m}$$

It can be seen from the formula that the least probable thickness of a cast-in-place T-girder railway bridge deck is (3/20) = 0.15 m. Thus, the 2.6-m deck thickness is 0.28 m.

Now, the last input data needed for the development of the iteration is the effective depth of the deck under consideration. The effective depth of a flexural member is the distance from the top of the extreme compression fiber to the centroid of tension reinforcement. The code permits the concrete cover thickness of 50 mm.

Effective depth (d) = Total depth of deck (t_{deck}) – concrete cover (cc) –half diameter of tension bar (0.5Ø)

$$d = t - (cc + 0.5\emptyset)$$
$$d = 0.28 - \left(0.05 + 0.5\left(\frac{25}{25}\right)\right)$$
$$d = 0.2173 \text{ m}$$

$$d = 2.173 \text{ mm}$$

The steel ratio of any section (ρ) is the amount of steel (i.e., structurally referred to as the area of steel) present in a section to the entire effective area $(bd)$of that section in a flexural member. As stated in the abstract, excessive use of rebars may have no importance on the design. Considering only the effective area of a flexural member, the steel ratio can be represented as

$$\rho_o = \frac{A_s}{bd}$$

In the actual analysis wherein material properties, the properties/dimensions of the section under analysis, the moment resulting from the applied loads and the strength reduction factors are taken into consideration to achieving the actual steel ratio relative to all of the above, the nominal expression becomes:

$$\rho_o = \left[1 - \sqrt{1 - \left(\frac{2M_{sd}}{\varphi\emptyset bd^2}\right)}\right] * \frac{\varphi f_c'}{f_y}$$

Substituting the values into the equation, relative to the provisions of (AREMA, 1999) for the strength reduction factors, the equation becomes:

$$\rho_o = \left[1 - \sqrt{1 - \left(\frac{2 * (8.9208)}{0.9 * 0.85 * 35 * 1.0 * 0.2173^2}\right)}\right] * \frac{0.85 * 35}{420}$$

$$\rho_o = [1 - 0.9930] * 0.0708$$

$$\rho_o = 0.0005$$

2.3 The Iteration

Now, consider the outcome of therho (ρ) equation above,

$$\rho_0 = [1 - 0.9930] * 0.0708$$

The righthand side consists of three interconnected dimensionless parts:

- The constant term (c): that is the '1"
- The original moment part (k_{mo}) that is the "0.9930", and
- The omega (ω)term, that is the "0.0708).

There are few quick points that can be concluded on from the equation:

- That the maximum value ρ_0 that can be attained is ωwhich is 0.0708 or roughly 7%
- That k_{mo} should always be less than c for unsupported RC spans
- That with adjustment made in the original moment term (k_{mo}), $\rho_0$would be adjusted as well
- Except the concrete and steel strengths are adjusted, ω is constant.

Because every code has a reinforcement ratio limit, it can be adjusted from here so that nominal steel ratio obtained can be in conformity with the code.This so done if the reinforcement ratio is structurally unacceptable or has to be adjusted to fit specific site conditions to include but not limited to architectural, economic, environmental, seismic, etc. In this light, the arbitrary adjusting factor (k_{adj}) is introduced. The $k_{adj}$to be used in this iteration is solely associated with directly adjusting the original moment term. Note also that the original moment term can also be discretized and another can be carried out withing. The magnitude of the factor is solelyto the discretion of the design engineer. However, the smoother the factor, the more accurate the result needed.The expression for the arbitrary adjusting factor is then coined as:

$$k_{adj} = \frac{k}{\left(\frac{S}{N}\right)}$$

where k is the first (least) number in the sequence and S/N is the last (greatest) number in the series. The series is the inclusive boundary from which the design engineer can choose from.For instance, an engineer may choose from one to one hundred (1 ~ 100). But for the purpose of this study, the range is from one to fifty-one (1 ~ 51) inclusive. Thus, the k_{adj}becomes

$$k_{adj} = \frac{1}{51} = 0.0196$$

The table below shows the various elements involved in the development of the iteration module. Nomenclatures and abbreviations herein are my personalthoughts; hence another engineer/author may modify whatsoever that is in here to suit his/her thoughts as well.

S/N	Const (c)	Origin. m-term (k_{mo})	Balance Factor [k_b]	Adjusted m-term [k_{ma}]	Stress Reduct. Factor [ω]	ρ_o adjust. by k_{ma} [ρ_a]	Origin. steel ratio [ρ_o]	ρ_{min} (Adopted ρ) [ρ]	Per. of k_b [$\%k_b$]	Rel. accu. of K_{ma} to K_{mo} [$\%k_{ma-mo}$]
1	1	0.9930	1.0000	0.9930	0.0708	0.00050	0.0005	0.0035	0.00	100.00
2	1	0.9930	0.9804	0.9736	0.0708	0.00187	0.0005	0.0035	1.96	98.05
3	1	0.9930	0.9608	0.9541	0.0708	0.00325	0.0005	0.0035	3.92	96.08
4	1	0.9930	0.9412	0.9347	0.0708	0.00462	0.0005	0.0035	5.88	94.13
5	1	0.9930	0.9216	0.9152	0.0708	0.00600	0.0005	0.0035	7.84	92.17
6	1	0.9930	0.9020	0.8957	0.0708	0.00738	0.0005	0.0035	9.80	90.20
7	1	0.9930	0.8824	0.8763	0.0708	0.00876	0.0005	0.0035	11.76	88.25
8	1	0.9930	0.8628	0.8568	0.0708	0.01014	0.0005	0.0035	13.72	86.28
9	1	0.9930	0.8432	0.8373	0.0708	0.01152	0.0005	0.0035	15.68	84.32
10	1	0.9930	0.8236	0.8179	0.0708	0.01289	0.0005	0.0035	17.64	82.37
11	1	0.9930	0.8040	0.7984	0.0708	0.01427	0.0005	0.0035	19.60	80.40
12	1	0.9930	0.7844	0.7790	0.0708	0.01565	0.0005	0.0035	21.56	78.45
13	1	0.9930	0.7648	0.7595	0.0708	0.01703	0.0005	0.0035	23.52	76.49
14	1	0.9930	0.7452	0.7400	0.0708	0.01841	0.0005	0.0035	25.48	74.52
15	1	0.9930	0.7256	0.7206	0.0708	0.01978	0.0005	0.0035	27.44	72.57
16	1	0.9930	0.7060	0.7011	0.0708	0.02116	0.0005	0.0035	29.40	70.60
17	1	0.9930	0.6864	0.6816	0.0708	0.02254	0.0005	0.0035	31.36	68.64
18	1	0.9930	0.6668	0.6622	0.0708	0.02392	0.0005	0.0035	33.32	66.69
19	1	0.9930	0.6472	0.6427	0.0708	0.02530	0.0005	0.0035	35.28	64.72
20	1	0.9930	0.6276	0.6233	0.0708	0.02667	0.0005	0.0035	37.24	62.77
21	1	0.9930	0.6080	0.6038	0.0708	0.02805	0.0005	0.0035	39.20	60.81
22	1	0.9930	0.5884	0.5843	0.0708	0.02943	0.0005	0.0035	41.16	58.84
23	1	0.9930	0.5688	0.5649	0.0708	0.03081	0.0005	0.0035	43.12	56.89
24	1	0.9930	0.5492	0.5454	0.0708	0.03219	0.0005	0.0035	45.08	54.92
25	1	0.9930	0.5296	0.5259	0.0708	0.03357	0.0005	0.0035	47.04	52.96
26	1	0.9930	0.5100	0.5065	0.0708	0.03494	0.0005	0.0035	49.00	51.01
27	1	0.9930	0.4904	0.4870	0.0708	0.03632	0.0005	0.0035	50.96	49.04
28	1	0.9930	0.4708	0.4676	0.0708	0.03769	0.0005	0.0035	52.92	47.09
29	1	0.9930	0.4512	0.4481	0.0708	0.03907	0.0005	0.0035	54.88	45.13
30	1	0.9930	0.4316	0.4286	0.0708	0.04046	0.0005	0.0035	56.84	43.16
31	1	0.9930	0.4120	0.4092	0.0708	0.04183	0.0005	0.0035	58.80	41.21
32	1	0.9930	0.3924	0.3897	0.0708	0.04321	0.0005	0.0035	60.76	39.24
33	1	0.9930	0.3728	0.3702	0.0708	0.04459	0.0005	0.0035	62.72	37.28
34	1	0.9930	0.3532	0.3508	0.0708	0.04596	0.0005	0.0035	64.68	35.33
35	1	0.9930	0.3336	0.3313	0.0708	0.04734	0.0005	0.0035	66.64	33.36
36	1	0.9930	0.3140	0.3119	0.0708	0.04872	0.0005	0.0035	68.60	31.41
37	1	0.9930	0.2944	0.2924	0.0708	0.05010	0.0005	0.0035	70.56	29.45
38	1	0.9930	0.2748	0.2729	0.0708	0.05148	0.0005	0.0035	72.52	27.48
39	1	0.9930	0.2552	0.2535	0.0708	0.05285	0.0005	0.0035	74.48	25.53
40	1	0.9930	0.2356	0.2340	0.0708	0.05423	0.0005	0.0035	76.44	23.56
41	1	0.9930	0.2160	0.2145	0.0708	0.05561	0.0005	0.0035	78.40	21.60
42	1	0.9930	0.1964	0.1951	0.0708	0.05699	0.0005	0.0035	80.36	19.65
43	1	0.9930	0.1768	0.1756	0.0708	0.05837	0.0005	0.0035	82.32	17.68
44	1	0.9930	0.1572	0.1561	0.0708	0.05975	0.0005	0.0035	84.28	15.72
45	1	0.9930	0.1376	0.1367	0.0708	0.06112	0.0005	0.0035	86.24	13.77
46	1	0.9930	0.1180	0.1172	0.0708	0.06250	0.0005	0.0035	88.20	11.80
47	1	0.9930	0.0984	0.0978	0.0708	0.06388	0.0005	0.0035	90.16	9.85
48	1	0.9930	0.0788	0.0783	0.0708	0.06526	0.0005	0.0035	92.12	7.89
49	1	0.9930	0.0592	0.0588	0.0708	0.06664	0.0005	0.0035	94.08	5.92
50	1	0.9930	0.0396	0.0394	0.0708	0.06801	0.0005	0.0035	96.04	3.97
51	1	0.9930	0.0200	0.0199	0.0708	0.06939	0.0005	0.0035	98.00	2.00

Table 4: The iteration table

Dissertation of Nomenclature

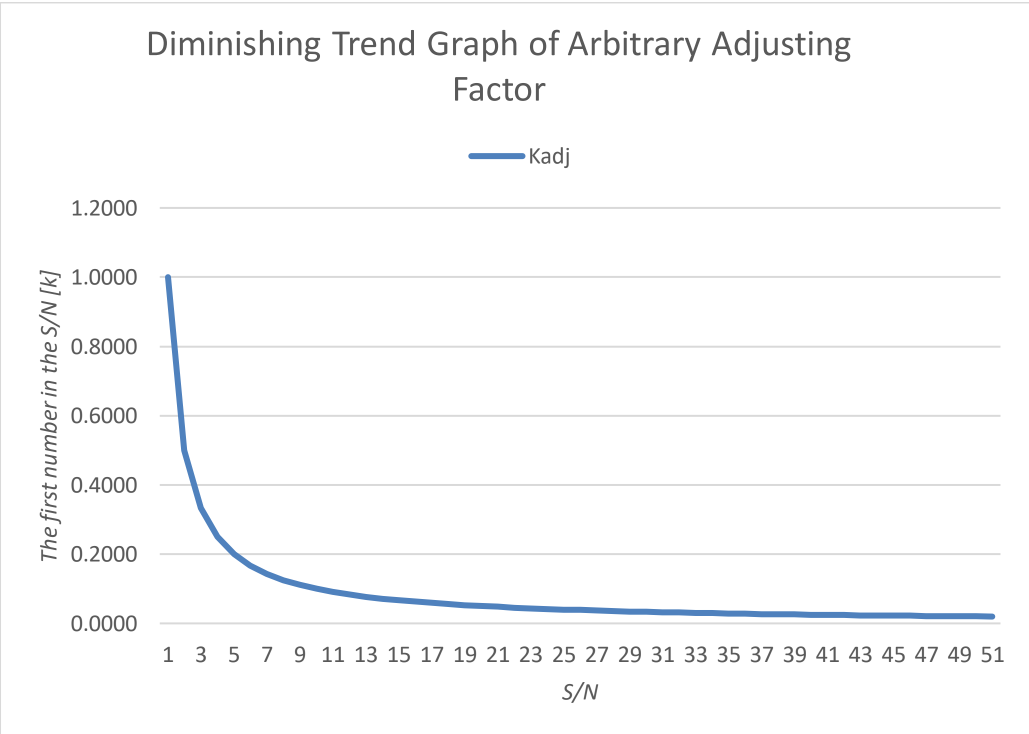
I have personally developed this table to enable rigorous iteration process that would enable designers manipulate all terms within it to suit a specified need.

1. K_{adj} ~ An arbitrary adjusting factor by which a moment term is adjusted
2. S/N ~ Sequence number in line with the kadj
3. Const (c)~ The constant value (=1) in the ϱequation
4. Origin. m term (k_{mo}) ~ The original moment term in the ϱ equation
5. Balance Factor [k_b] ~ A factor by which K_{adj} is accumulatedly subtracted from each moment term
6. Adjusted m-term [k_{ma}] ~ The new moment term after the application of the balance factor
7. Stress Reduct. Factor [ω] ~ A stress reduction factor for flexural members
8. ϱo adj'd by k_{ma} [ϱa] ~ The new actual ϱ value after the application of the balance factor
9. Origin. steel ratio [ϱo] ~ The Original steel ratio value of the section obtained without any adjustment to dimensional and material properties
10. ρ_{min} (Adopted ρ) [ρ] ~ The minimum code-required ρ value of the section adopted for the design because of the inadequacy of the calculated ρ
11. Per. of kb [%k_b] ~ Percentage by which the arbitrary adjusting factor has caused the balance factor to be reduced
12. Rel. accur. of K_{ma} to K_{mo} (%k_{ma-mo}) ~ The Relative Accuracy of K_{ma} to K_{mo} is the ratio of the adjusted moment term to the original moment term, multiplied by 100.

To effectively understand the table, a further deeper analysis of each nomenclature is considered either individually or in a group.

2.3.1 K_{adj} and k vs. S/N

In this work, the arbitrary adjusting factor is a very key mathematical computing tool used to produce these tabulated results. To better fine-tune the K_{adj}, it dictates increasing the occur sequence number as high as possible. The inverse relationship created between Kadj and S/N can graphically be represented as below:



Iterative graph 1: View of the relationship between k and S/N (=k_{adj})

It can now be visualized that the function is inversely proportional and that as S/N approaches positive infinity, the K_{adj} approaches zero. Thus,it can be safely hypothesized that:

K_{adj} equals zero as S/N approaches positive infinity

The accumulated k_{adj} which is used once S/N is greater than k, at any value of S/N (i.e., k_{adj-a}) can be obtained as an integral function

$$k_{adj\_a} = \int_{k}^{S/N} k_{adj} dx$$

2.3.2Const (c) and Origin. m term (k_{mo})

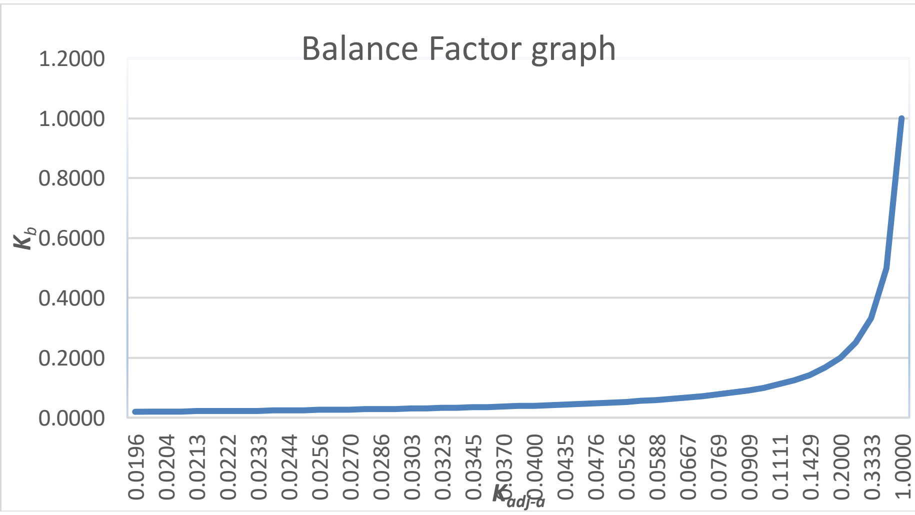
The constant k (which = 1) is a derived figure in the original equation. A noticeable characteristic of the constant term is that it dictates the moment term to be less than unity. If unity, ρ_o is automatically zero, thus depicting a condition for the use of plain concrete during construction. Cases of such would occur when the span of the member is very short compared to its depth, or probably for on-grade sections. Should the original moment term exceed unity, the value of ρobecomes negative. There are four contributing factors that may lead to a negative ρo instance.

- When the design moment is very large
- When the width of the section is very small
- When the depth of the section is very small
- When the compressive strength of the concrete is very low

A negative ρ_o value had probably not been adopted in design. If encountered, probably because of the large moment capacity, it's logical to adjust one or more of the values of conditions a, b, c and or d and recalculate.

2.3.3 Balance Factor [k_b]

A factor by which K_{adj} is accumulatedly subtracted (K_{adj-a}) from the original moment term. The balance factor is designed to enable a reduction in the original moment term leading to zero. This is so done to estimate a corresponding value of the moment term that might adequately match the adopted ρ value of the section. From the onset (that is, at S/N = 1), all parameters are unchanged and are as written in the original ρ equation.



Iterative graph 2: View of the relationship between k_b and k_{adj}

Mathematically, the statement can be represented as:

$$K_{bo} = \frac{k_{mo}}{k_{mo}} = 1$$

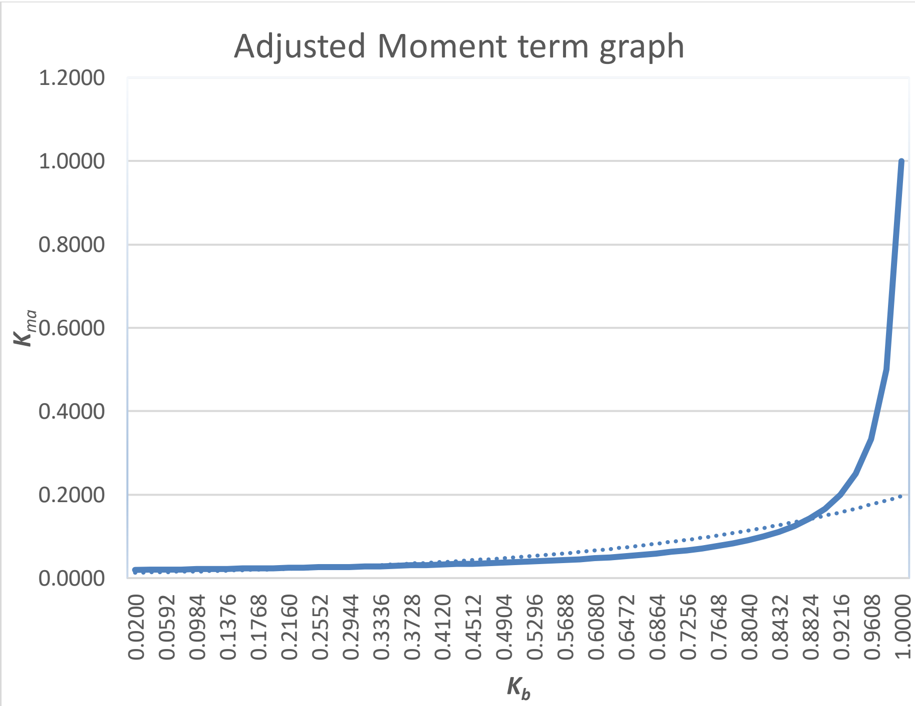
…where k_{bo} is the initial k_b obtained by dividing the original moment term by it self. Thus,

$$K_b = 1 - kadj\_a$$

$$k_b = 1 - \int_k^{S/N} k_{adj} dx$$

2.3.4 Adjusted m-term [k_{ma}]

The shape of the graph of k_{ma} is as same as that of k_b because of the direct relationship that exists between the both.



Iterative graph 3: Relationship between $k_{ma}$and k_b

Here, as k_b approaches maximum, adjusted moment approaches original moment. Mathematically, the expression can be derived.

$$K_{ma} = K_{mo} \times K_b$$

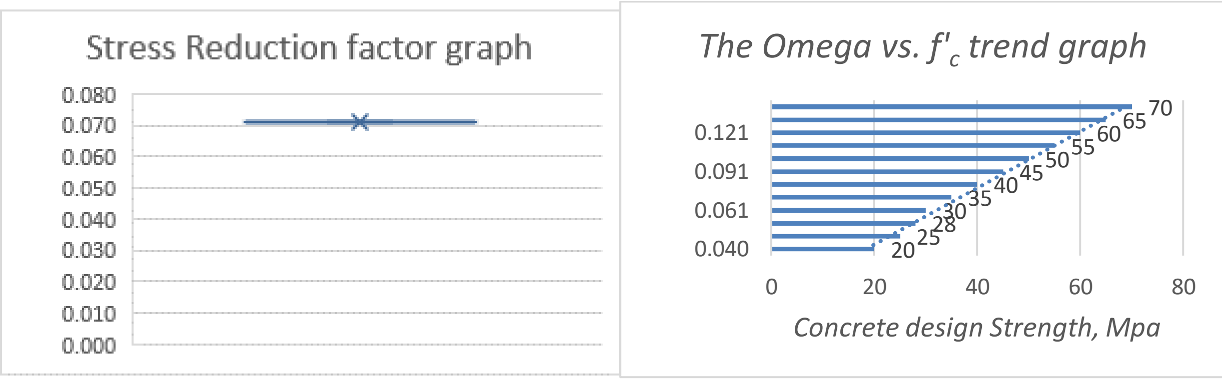
$$= K_{mo} \times (K_{bo}\ \_\ ka_{dj}\ \_\ a)$$

Involving integrated calculus, the above expression can be rewritten as

$$k_{ma} = k_{mo}(1 - \int_k^{S/N} k_{adj} dx)$$

2.3.5 Stress Reduct. Factor [ω]

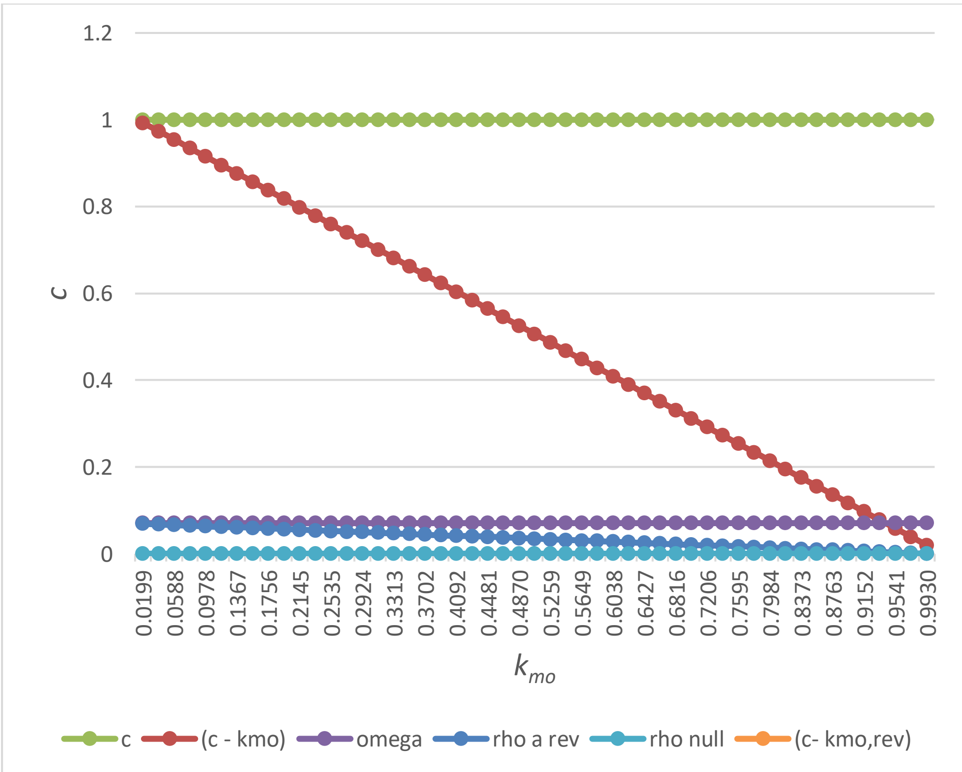
The strength reduction factor is a ratio of concrete compressive strength to that of the yield strength of the reinforcing steel interacting with a specified capacity-reduction factor. It is constant throughout for the same concrete and reinforcement properties. Should these material properties change, the value would also change.



Iterative graph 4: The ω graph

2.3.6 The c, k_{mo}, ω, k_{ma}, ρ_o relationship

As previously stated, the original moment term should always be less than unity to avoid obtaining a negative original steel ratio. The relationship between the original moment, the constant, the stress reduction factor, the original steel ratio and the adjusted steel ratio due to the effect of the adjusted moment, is graphically given in the table below. With the constant c being the between major determining coordinate, all values are aligned to it. As c is constant; it lies top of the vertical axis at a zero-degree slope. The highest value of the original moment is seen approximately equal to the constant and as it decreases, (c-k_{mo}) increases thus concomitantly increasing ρ_a in a very little manner.



Iterative graph 5: Relationship between c, k_{mo}, ω, k_{ma} and ρ_o

It is previously derived that the expression for the adjusted moment term is:

$$k_{ma} = k_{mo}(1 - \int_{k}^{S/N} k_{adj}dx)$$

…but

$$\rho_a = (c - K_{ma})$$

Thus, substituting for k_{ma}, the resulting expression yields

$$\rho_a = (c - [k_{mo}(1 - \int_{k}^{S/N} k_{adj}dx)])\,\omega$$

However, with the original steel ratio readily available in the equation, a manipulation in the formula leads to the following derived formula:

$$\rho_o = (c - K_{mo})$$

$$\rho_o = (c - \frac{k_{ma}}{(1 - k_{adj\,\_\,a})})\omega$$

$$\rho_o = (c - \frac{k_{ma}}{\left(1 - \int_{k}^{\frac{S}{N}} K_{adj}dx\right)})\omega$$

The gradient of the change in ρ_a, which shows how steady the graph increases can be formulated as given below:

$$\Delta\rho_a = \frac{\Delta H}{\Delta L} = \frac{\Delta\rho_a}{\Delta k_{ma}}$$

$$\Delta\rho_a = \frac{\Delta\{\left(c - \left[k_{mo}\left(1 - \int_{k}^{\frac{S}{N}} k_{adj}dx\right)\right]\right)\omega\}}{\Delta(k_{mo}(1 - \int_{k}^{S/N} k_{adj}dx))}$$

$$\Delta\rho_a = \frac{\left\{\left(c - \left[k_{mo}\left(1 - \int_{k}^{\frac{S}{N}} k_{adj}dx\right)\right]\right)\omega\right\} - \left\{\left(c - \left[k_{mo}\left(1 - \int_{k}^{k} k_{adj}dx\right)\right]\right)\omega\right\}}{\{k_{mo}(1 - \int_{k}^{S/N} k_{adj}dx)\} - \{k_{mo}(1 - \int_{k}^{k} k_{adj}dx)\}}$$

$$\Delta\rho_a = \frac{\left\{\left(\omega c - \omega\left[k_{mo}\left(1 - \int_{k}^{\frac{S}{N}} k_{adj}dx\right)\right]\right)\right\} - \left\{\left(\omega c - \omega\left[k_{mo}\left(1 - \int_{k}^{k} k_{adj}dx\right)\right]\right)\right\}}{\{k_{mo}(1 - \int_{k}^{S/N} k_{adj}dx)\} - \{k_{mo}(1 - \int_{k}^{k} k_{adj}dx)\}}$$

$$\Delta\rho_a = \frac{\omega k_{mo}(\int_{k}^{S/N} k_{adj}dx - \int_{k}^{k} k_{adj}dx)}{-k_{mo}(\int_{k}^{\frac{S}{N}} k_{adj}dx - \int_{k}^{k} k_{adj}dx)}$$

$$\Delta\rho_a = \frac{\omega k_{mo}}{-k_{mo}}$$

$$\Delta\rho_a = -\omega$$

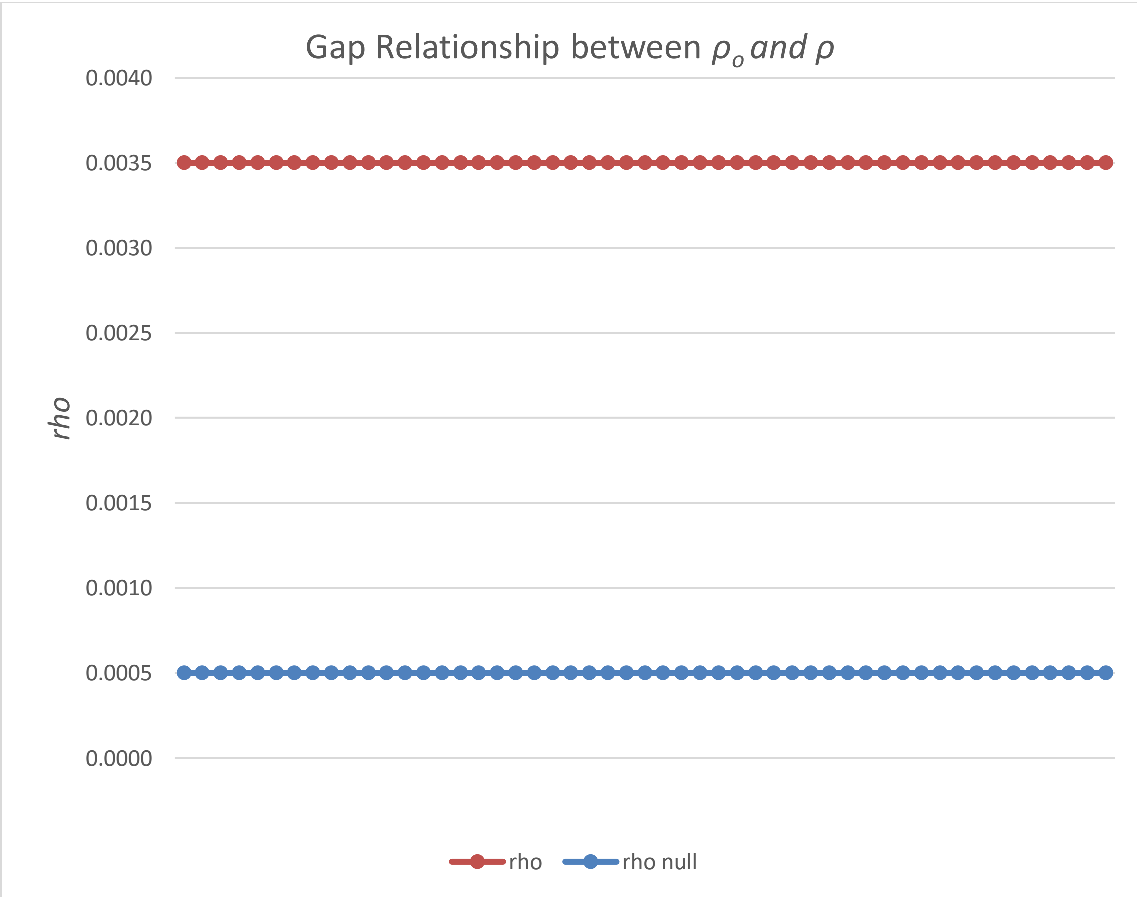
Because of the complexity of this equation, a proof would be welcoming. Thus, prove that the expression is exact!

$$\Delta\rho a = \frac{\Delta\rho_a}{\Delta k_{ma}} = \frac{\rho_{a51} - \rho_{a1}}{k_{ma51} - k_{ma1}} = \frac{(0.06939 - 0.00050)}{0.0199 - 0.9930} = \frac{0.06889}{-0.9731} = -0.0708 = -\omega, \ldots Correct$$

Why is the slope negative? The slope is negative because as seen in the table, there is an inverse relationship between the original moment term and that of the adjusted rho (ρ_a). As the original moment is decreased by the effect of the balancing factor, the adjusted rho increases gradually.

2.3.7 ρo, ρ, %k_b, and $k_{ma\text{-}mo}$ Relationship

The two tabulated steel ratios ρo and ρ are independent values of the various iterations done in the table. However, the gap between the two values can be graphically measured and a mathematical statement can be drawn.



Iterative graph 6: Relationship between ρ_o and ρ

As it has previously been derived that

$$\rho_o = \left(c - \frac{k_{ma}}{\left(1 - \int_k^{\frac{S}{N}} K_{adj} dx\right)}\right)\omega$$

the gap between the design steel ratio and that of the minimum steel ratio is formulated as below. The adoption of the absolute value dictates that the larger of the two shall be adopted as the design ratio.

$$\rho_g = |\rho - \rho_o|$$

$$\rho_g = \left|\rho - \left\{\left(c - \frac{k_{ma}}{(1 - \int_k^{S/N} k_{adj} dx)}\right)\omega\right\}\right|$$

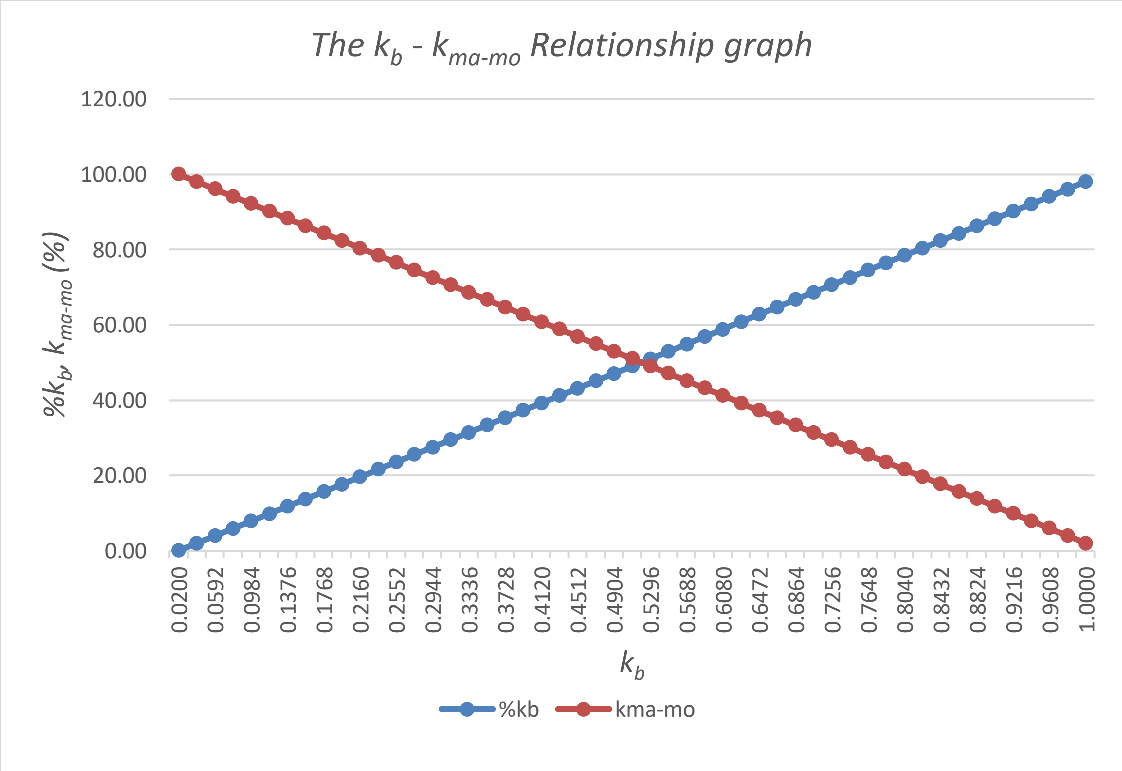
Also, because of the complexity of this equation, a proof would be welcoming. Thus, prove that the expression is exact!

$$\rho_g = |\rho - \rho_a| = |0.0035 - 0.0005| = |0.003|$$

…and using the integrated equation,

$$\rho_g = \left|\rho - \{(c - \frac{k_{ma}}{(1 - \int_k^{S/N} k_{adj} dx)})\omega\}\right| = \left|0.0035 - \left\{\left(1 - \frac{0.0199}{1 - \int_1^{51} 0.0196 dx}\right) * 0.0708\right\}\right| = 0.00315 \approx 0.003, \mathbf{OK}$$

$\%k_b$ shows the trend of reduction in the balance factor through to the last number in the series. This means thatthe percentage would always increase because of the accumulation. On the other hand, as k_b decreases, $k_{ma\text{-}mo}$ decreases. The relationship is so fashioned because as k_b decreases, k_{ma} also decreases. The most important point to note is that the graphs intersect. The point of intersection also coincides with the point of intersection of k_b and k_{ma}.



Iterative graph 7: The $\%k_b$ and $k_{ma\text{-}mo}$relationship

With previous derivations carried out, $\%k_b$ can be represented by:

$$\%k_b = (k_{bo} - k_b) \times 100$$

…and in a more detailed and integrational calculus form,

$$\%k_b = \{1 - (1 - \int_k^{S/N} k_{adj} dx)\} \times 100$$

On the other hand, $k_{ma\text{-}mo}$ can be represented as:

$$\%k_{ma\_mo} = \frac{k_{ma}}{k_{mo}} \times 100$$

…wherein the ratio k_{ma} to k_{mo} can be manipulated from

$$k_{ma} = k_{mo}(1 - \int_{k}^{S/N} k_{adj} dx)$$

$$\frac{k_{ma}}{k_{mo}} = (1 - \int_{k}^{S/N} kadj\, dx)$$

Thus $\%k_{ma\text{-}mo}$ equals,

$$\%kma\_mo = (1 - \int_{k}^{S/N} k_{adj} dx) \times 100$$

The process of developing the iteration ends with the above expression.

3 Findings

The thematic findings of this paper are presented in section "2.3 The Iteration". Yet then still, from the iteration table, a general statement can be drawn:

- ✓ That at S/N = 1, all values are constant
- ✓ That at around 95% of $\%k_{ma\text{-}mo}$, ρa corresponds to ρ unlimited the range of series (S/N).

However, depending on the materials properties, f'_c and f_y, the following conclusions can be drawn as well:

- ➢ K_{ma} is about 99% of k_b
- ➢ In hundredth, k_{ma} equals k_b at alternating distances
- ➢ Around the bottom 5% of $\%k_{ma\text{-}mo}$, ρa is approximately equal to ω

4 Conclusion and Recommendations

Other than the original steel ratio equation used in the development of the iteration, all other equations were developed by me as a scholarly work to adjusting any parameter the designer may foresee prudent. In subsequent publications, it shall also be proven that at certain known materials properties, the moment capacity of a section solely depends on the effective depth of the member. The work presented herein is solely based on dead load only and that other researchers can include other modes of loading as well as other types of structures.

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