An EOQ Model for Deteriorating Items with Time –Varying Demand and Time-Dependent Holding Cost without Shortages.

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Abstract – An optimal policy for deteriorating item is developed in this paper. The deterioration of an item is constant with time-varying demand and the holding cost is assumed to be time- dependent. The aims of the model are to minimize the total average cost by finding the optimal cycle time and ordering quantity. Shortages are not allowed in the model. The Mathematical model is developed by the use of differential equations to get the optimal value. A simple to understand solution procedure is provided to establish the policy proposed. It is discovered that the effect of time-varying demand rate and time dependent holding cost on deterioration of an items cannot be easily ignored in the management of inventory. Numerical example with sensitivity analysis are given to affirm the model developed and procedure use for the solution

Keywords: Inventory System, Shortage, Deterioration, Holding cost

1. Introduction

In every business organization, trade and industry, proper management of inventories is very important. This is essential so as to avoid a breakdown in their daily affairs. Basically, we experience inventories in almost every area of life. In business world, company dealing with physical products, including manufacturers, wholesalers, retailer's needs proper management of inventories. The problems of inventories arise when there is need to make an optimal decision in order to minimize the total cost or maximize the total profits of the inventory system. This decision of the inventory problem deals majorly on when should the system be replenished and how much should be added to the system.

Deterioration of an item has become a very indispensable phenomenon in the management of inventory problem over the past few decades. Items deteriorate and become useless such that it cannot be used again for the purpose it is intended for. Therefore, proper planning must be put in place to maintain the inventory economically so as to yield more revenues for the industry/organization that keeps inventory.

A deterioration process of an item is defined as decay, damage, obsolescence and loss of values in a product as time progresses. Deteriorating items can be grouped into two major categories. The first category involves items that becomes decay, damaged, evaporate, expired, devalued as a result of passage of time, Examples of such items includes meats, fruits, vegetables, medicine, films, food, pharmaceuticals etc.

The second category consists of items that partly or absolutely lost its value through time as a result of new technology or the introduction of another alternative. Examples of items in this category are computer chips, mobile phone or hand set, clothes, electronics, fashion and other seasonal goods and so on. Short life cycle is peculiar to both categories.

The classical inventory model as the economic order quantity (EOQ) for the deteriorating items was first proposed by Harris [1] who stated that the inventory is depleted majorly due to the constant demand rate. Since then, many researchers have studied inventory model for deteriorating items.Whitin and Hadley [2] analyzed a study on fashion goods that deteriorate at the close of storage period. Later, Ghare and Schrader [3] presented an inventory model for good that decay exponentially. Also, Shah and Jaiswal [4] studied a policy for an order- level inventory model with constant rate of deterioration. An inventory model for a deteriorating item for a linearly time dependent demand rate and constant replenishment was presented by Dave and Patel [5]. Ata et al [6] established a deteriorating vendor managed inventory model for a two –level supply chain. The models consist of one vendor and many non- competing retailers with different deteriorating rate for raw material and finished product. No shortage is considered. In addition, Singh et al [7] presented an optimal inventory policy model for deteriorating items with time dependent deterioration rate and different demand rate without shortages. Furthermore, Rangarajan and Karthikeyan [8] proposed an EOQ deteriorating inventory model for items that has different demand rate without shortages. No shortage is considered for the above mentioned model.

Elsayed and Teresi [9] developed two deterioration inventory models for an economic order quantity. The first model considered the demand to be deterministic with finite rate of production without shortage while the second model considered the demand to be a random variable with two parameter Weibull distribution deterioration with shortages allowed. Furthermore, Peter et al. [10] presented a note on deteriorating inventory model for items with ramp type demand rate. They discovered some errors in the previous published works on ramp type demand models and provided a precise and effective method of obtaining the optimal solution. Sanni and Chukwu [11] proposed a deterministic inventory model for deteriorating items with three parameter Weibull distribution, having ramp-type demand. The model was developed with shortages which are backlogged completely

A detailed review of literatures on inventory models for deteriorating items are given by Nahmias [12], Raafat [13], Goyal and Giri [14], Bakker et al [15], Janssen et al. [16].

It is frequently assumed in the basic economic order quantity model that the demand of a product is constant, but in reality, this may not be true all the time because customers demand do varies with time. Thus, the study of inventory model with time- varying demand is very appropriate in that situation. This type of time varying demand model is of important because it helps to model appropriately the behaviors and the progression of the inventory.

Many research works has been done in this direction. Donaldson [17] studied the classical no shortage replenishment inventory model for a demand pattern having a linear trend .Later, Silver [18] proposed a deterministic inventory model with demand following a linear trend. The objective of the model was to choose timing and replenishment sizes so as to reduce the total of carrying and replenishment cost to the lowest. In addition, Benkherouf and Mahmoud [19] proposed a replenishment inventory model for deteriorating items with constant rate of deterioration and time-varying demand considering shortages. Also, Goswami and Chaudhuri [20] proposed an economic order quantity (EOQ) model with linear trend in demand, assuming shortages. Furthermore, Zhao et al [21] formulated a heuristic replenishment policy with demand having a linear decreasing trend. A deteriorating inventory model for items with constant rate of deterioration and time-varying demand with shortages was presented by Sicilia et al. [22].

Holding cost has been considered as a constant function in most of the inventory model discussed so far. However, in reality, holding cost is not always a constant function. For instant, if perishable and deteriorating items such as food items are kept in storage, the storage facilities and services needed will equally be more sophisticated to preserve the items. This will definitely lead to higher holding cost. These imply that holding cost varying with passage of time; therefore, holding cost is not always a constant function.

In this direction, Goh [23] proposed a deterministic inventory model with general demand and time varying holding cost. Also, Alfred [24] analyzed an inventory model having demand rate dependent on stock–level and holding cost varies with time. Mishra [25] analyzed a deteriorating inventory model for items with Weibull distribution deterioration and variable holding cost considering salvage value and shortages. Singh et al [26] proposed a deteriorating inventory model for items with time dependent holding cost. Shortages are not considered in the model. Additionally, Dutta and Pavan [27] derived an optimal inventory model for items with fuzziness in demand, ordering cost and holding cost without shortages.Tripathi [28] examined an inventory model for deteriorating items that have different holding cost and different demand rate. Moreover, Palaniel and Uthayakumar [29] presented a deteriorating inventory model for non-instantaneous deteriorating items having demand as power pattern, holding cost is dependent on time with permissible delay in payment and shortages are considered and partially backlogged. An inventory and pricing model with demand dependent on price and variable holding cost and quantity discount was discussed by Alfares and Ghaithan [30]. Later, the same authors [31] developed a review papers under variable holding cost for both EOQ and EPQ models.

In this present work, a deteriorating inventory model is proposed with a constant rate of deterioration. The demand rate follows a power pattern and the holding cost is considered linear functions of time as against the general assumption that the holding cost is constant. This is so because holding cost is usually connected with storage of an item until usage or stored inventory. It entails various cost components such as protection against thief, insurance, capital tied up etc. In addition, holding cost can be viewed as a function of combined factors such as optimal quantity stored during a period and can be accessed either continuously or periodically. Since holding cost depends on many factors, it can be represented as a linear function of time. A shortage is not allowed in this model. Numerical example is given to establish the model and sensitivity analysis of the optimal solution in relation to the major parameters in the model are discussed.

The rest of the paper is arranged in the following way: Assumptions and the notation used in the paper is given in section 2, follow by mathematical formulation and solution procedure in section 3.Numerical example and sensitivity analysis is provided in section 4 and 5.The paper ends with conclusion and future directions in section 6.

2. Assumptions and Notation

The development of the model is based on the following assumptions:

- The demand rate of the items is represented by power pattern and is a continuous function of time
- Lead time is negligible
- The deterioration rate of items is constant
- Shortages are not allowed
- The rate of replenishment is infinite
- Planning horizon is finite

The following notation will be use throughout the paper.

- T Length of inventory cycle (time)
- Q Order quantity/units
- I(t) Inventory level at time t
- r Average demand per inventory cycle.
- n Demand pattern index (n is non-negative)
- K Ordering cost (₦/order)
- h holding cost per unit/₦/time
- C Cost per deteriorated unit (₦/order)
- SC Set up cost/cycle
- DC Deteriorating cost/time units/cycle
- HC Holding cost is a linear function of time (t) = a +b t, (a and b are non-negative)
- TC Total cost of the inventory model/time units

3. Mathematical formulation of the model

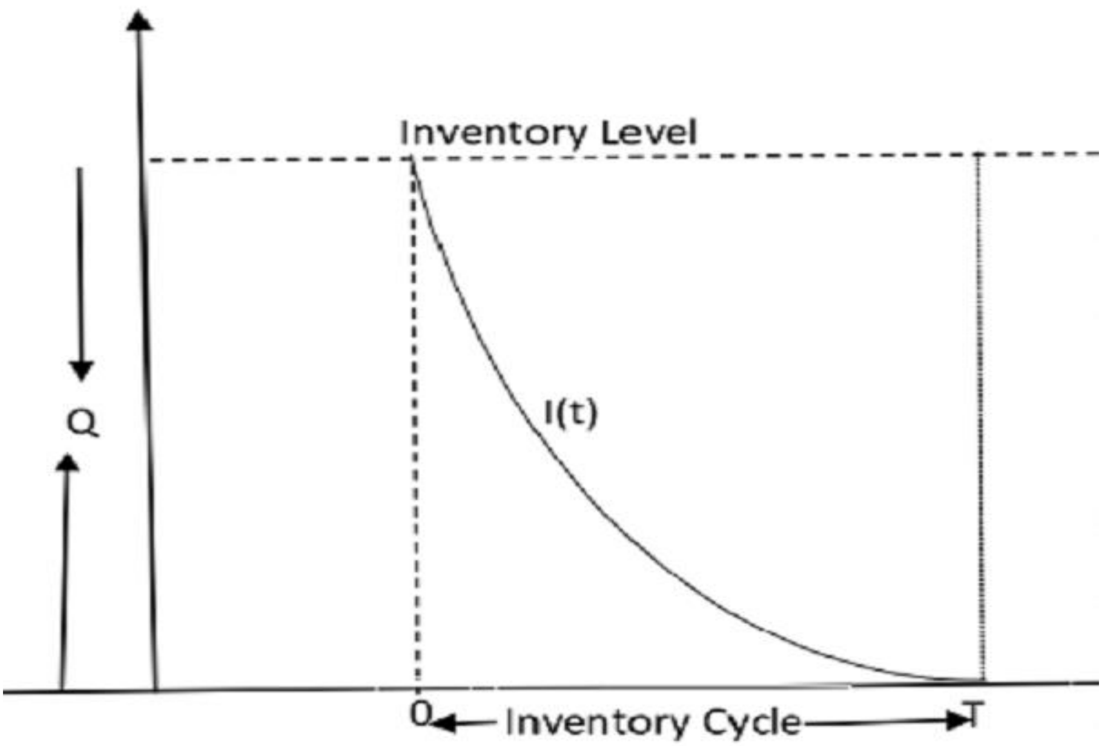


Figure 1: Inventory model with power demand pattern and deterioration of stored products

The immediate inventory model I(t) at any time t during the schedule period T is given by:

$$\frac{\partial I(t)}{\partial t} + \alpha I(t) = D(t) 0 \leq t \leq T \qquad (1)$$

Using integrating factor I.F.$=e^{\int pdt}$

$$\frac{\partial I(t)}{\partial t} e^{\alpha t} + \alpha I(t) e^{\alpha t} = -D(t) e^{\alpha t}$$

$$I'(t) e^{\alpha t} + \alpha I(t) e^{\alpha t} = -\frac{r\, t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}-1}} e^{\alpha t}$$

$$I(t) e^{\alpha t} = -\frac{r}{nT^{\frac{1}{n}-1}} [\int t^{\frac{1}{n}-1}\, e^{\alpha t} dt]$$

Using power series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \ldots\ldots\ldots\ldots\ldots + \frac{x^n}{n!}$

$$e^{\alpha t} = 1 + \alpha t + \frac{(\alpha t)^2}{2!} + \frac{(\alpha t)^3}{3!}$$

$$e^{\alpha t} = 1 + \alpha t + \frac{(\alpha t)^2}{2!} + \frac{(\alpha t)^3}{3!} + \cdots \ldots\ldots\ldots\ldots.. +$$

Since α is small, we truncate the series at the third term.

$$I(t) e^{\alpha t} = -\frac{r}{nT^{\frac{1}{n}-1}} \left[\int t^{\frac{1}{n}-1} dt + \int \alpha\, t^{\frac{1}{n}} dt + \frac{\alpha^2}{2} \int t^{\frac{1}{n}+1} dt \right]$$

$$I(t)e^{\alpha t} = -\frac{r}{nT^{\frac{1}{n}-1}}\left[nt^{\frac{1}{n}} + \frac{\alpha nt^{\frac{1}{n}+1}}{n+1} + \frac{\alpha^2 nt^{\frac{1}{n}+2}}{4n+1}\right] + C$$

$$I(t)e^{\alpha t} = -\frac{r}{T^{\frac{1}{n}-1}}\left[t^{\frac{1}{n}} + \frac{\alpha t^{\frac{1}{n}+1}}{n+1} + \frac{\alpha^2 t^{\frac{1}{n}+2}}{4n+1}\right] + C$$

Using boundary condition, I (T) = 0

$$0 = -\frac{r}{T^{\frac{1}{n}-1}}\left[T^{\frac{1}{n}} + \frac{\alpha T^{\frac{1}{n}+1}}{n+1} + \frac{\alpha^2 T^{\frac{1}{n}+2}}{4n+1}\right] + C$$

$$C = \frac{r}{T^{\frac{1}{n}-1}}\left[T^{\frac{1}{n}} + \frac{\alpha T^{\frac{1}{n}+1}}{n+1} + \frac{\alpha^2 T^{\frac{1}{n}+2}}{4n+1}\right]$$

$$I(t)e^{\alpha t} = \frac{r}{T^{\frac{1}{n}-1}}\left[\left(T^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\alpha}{n+1}\left(T^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right) + \frac{\alpha^2}{4n+1}\left(T^{\frac{1}{n}+2} - t^{\frac{1}{n}+1}\right)\right]$$

$$I(t) = \frac{re^{-\alpha t}}{T^{\frac{1}{n}-1}}\left[\left(T^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\alpha}{n+1}\left(T^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right) + \frac{\alpha^2}{4n+1}\left(T^{\frac{1}{n}+2} - t^{\frac{1}{n}+1}\right)\right] \tag{2}$$

$$0 \le t \le T$$

The optimum order quantity level is given by I (0) = Q

$$Q = \frac{r}{T^{\frac{1}{n}-1}}\left[T^{\frac{1}{n}} + \frac{\alpha}{n+1}T^{\frac{1}{n}+1} + \frac{\alpha^2}{4n+1}T^{\frac{1}{n}+2}\right] \tag{3}$$

The total cost (TC) per unit time consists of the following cost components:

a) The Holding cost HC per cycle [0, T] is given by:

$$HC = \frac{1}{T}\int_0^T (a+bt)I(t)dt$$

$$HC = \frac{1}{T}\int_0^T (a+bt)\left\{\frac{re^{-\alpha t}}{T^{\frac{1}{n}-1}}\left[\left(T^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\alpha}{n+1}\left(T^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right) + \frac{\alpha^2}{4n+1}\left(T^{\frac{1}{n}+2} - t^{\frac{1}{n}+1}\right)\right]\right\}\text{dt}$$

$$HC = \frac{1}{T}\int_0^T (a+bt)\left\{\frac{r(1-\alpha t)}{T^{\frac{1}{n}-1}}\left[\left(T^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\alpha}{n+1}\left(T^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right) + \frac{\alpha^2}{4n+1}\left(T^{\frac{1}{n}+2} - t^{\frac{1}{n}+1}\right)\right]\right\}$$

$$HC = \frac{r}{T^{\frac{1}{n}}}\int_0^T (a+bt)\left\{(1-\alpha t)\left(T^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\alpha(1-\alpha t)}{n+1}\left(T^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right) + \frac{\alpha^2(1-\alpha t)}{4n+2}(T^{\frac{1}{n}+2} - t^{\frac{1}{n}+2})\right\}$$

After integrating and simplification, we have:

$$HC = \frac{rh}{T^{\frac{1}{n}}}\left[aT^{\frac{1}{n}+1} - \frac{anT^{\frac{1}{n}+1}}{n+1} - \frac{\alpha anT^{\frac{1}{n}+2}}{n+1} + \frac{\alpha anT^{\frac{1}{n}+2}}{(2n+1)(n+1)} + \frac{\alpha^2 aT^{\frac{1}{n}+2}}{n+1} - \frac{\alpha^2 aT^{\frac{1}{n}+3}}{2(n+1)} + \frac{\alpha^2 anT^{\frac{1}{n}+3}}{(n+1)(3n+1)} \right.$$
$$+ \frac{\alpha^2 aT^{\frac{1}{n}+3}}{(4n+2)} - \frac{\alpha^2 naT^{\frac{1}{n}+3}}{(3n+1)(4n+3)} - \frac{\alpha^3 aT^{\frac{1}{n}+3}}{2(4n+2)} + \frac{\alpha^3 anT^{\frac{1}{n}+4}}{(4n+1)(4n+2)} + \frac{bT^{\frac{1}{n}+2}}{2} - \frac{bnT^{\frac{1}{n}+2}}{2n+1}$$
$$- \frac{b\alpha T^{\frac{1}{n}+3}}{3} + \frac{bn\alpha T^{\frac{1}{n}+3}}{(n+1)(3n+1)} + \frac{b\alpha T^{\frac{1}{n}+3}}{2(n+1)} - \frac{b\alpha^2 T^{\frac{1}{n}+4}}{3(n+1)} + \frac{bn\alpha^2 T^{\frac{1}{n}+4}}{(n+1)(4n+1)} + \frac{b\alpha^2 T^{\frac{1}{n}+4}}{2(2+4n)}$$
$$- \frac{bn\alpha^2 T^{\frac{1}{n}+4}}{(2+4n)(4n+1)} - \frac{b\alpha^3 T^{\frac{1}{n}+4}}{3(2+4n)}$$
$$\left. + \frac{bn\alpha^3 T^{\frac{1}{n}+5}}{(2+4n)(5n+1)}\right] \qquad (4)$$

The number of deteriorated unit during the cycle period [0.T] is given by:

$$NDU = Q - \int_0^T D(t)dt, where\ D(T) = \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}-1}}$$

$$= \frac{r}{T^{\frac{1}{n}-1}}\left[T^{\frac{1}{n}} + \frac{\alpha}{n+1}T^{\frac{1}{n}+1} + \frac{\alpha^2}{4n+1}T^{\frac{1}{n}+2}\right] - rT$$

$$= \frac{r\alpha T^2}{n+1} + \frac{r\alpha^2 T^3}{4n+2}$$

Therefore, the deteriorating cost is given by:

b) $DC = \frac{C_1}{T}(\frac{r\alpha T^2}{n+1} + \frac{r\alpha^2 T^3}{4n+2})$

$$= \frac{C_1 \alpha rT}{n+1} + \frac{C_1 r\alpha^2 T^2}{4n+2} \qquad (5)$$

c) Ordering cost /set up cost (SC) per cycle [0,T] is given by :

$$SC = \frac{K}{T} \qquad (6)$$

The total cost per unit time is given by TC =HC +DC +SC

$$TC = \left[harT - \frac{ahrnT}{n+1} - \frac{r\alpha hanT^2}{n+1} + \frac{hr\alpha anT^2}{(2n+1)(n+1)} + \frac{hr\alpha^2 aT^2}{n+1} - \frac{rh\alpha^2 aT^3}{2(n+1)} + \frac{hr\alpha^2 anT^3}{(n+1)(3n+1)} + \frac{rh\alpha^2 aT^3}{(4n+2)} - \frac{hr\alpha^2 naT^3}{(3n+1)(4n+3)} - \right.$$
$$\frac{rh\alpha^3 aT^3}{2(4n+2)} + \frac{rh\alpha^3 anT^4}{(4n+1)(4n+2)} + \frac{rhbT^2}{2} - \frac{rhbnT^2}{2n+1} - \frac{rhb\alpha T^3}{3} + \frac{hbrn\alpha T^3}{(n+1)(3n+1)} + \frac{bhr\alpha T^3}{2(n+1)} - \frac{brh\alpha^2 T^4}{3(n+1)} + \frac{brhn\alpha^2 T^4}{(n+1)(4n+1)} + \frac{brh\alpha^2 T^4}{2(2+4n)} -$$
$$\left.\frac{bhnr\alpha^2 T^4}{(2+4n)(4n+1)} - \frac{bhr\alpha^3 T^4}{3(2+4n)} + \frac{hbnr\alpha^3 T^5}{(2+4n)(5n+1)} + \frac{C_1 \alpha rT}{n+1} + \frac{C_1 r\alpha^2 T^2}{4n+2} + \frac{K}{T}\right] \qquad (7)$$

The necessary and sufficient condition for the total cost (TC) to be minimize is that

$$\frac{\delta(TC)}{\delta T}=0\ and\ \ \frac{\delta^2(TC)}{\delta T^2}>0\,,for\ all\ T>0 \tag{8}$$

$$\frac{\delta(TC)}{\delta T}=har-\frac{harn}{n+1}-\frac{2ahmr\alpha T}{n+1}+\frac{2hra\alpha nT}{(2n+1)(n+1)}+\frac{2hra\alpha T}{n+1}-\frac{3rha\alpha^2T^2}{(2n+2)}+\frac{3hranT^2\alpha^2}{(n+1)(3n+1)}+\frac{3hra\alpha^2T^2}{(4n+2)}-\frac{3hrana^2T^2}{(3n+1)(4n+3)}-\frac{3hra\alpha^3T^2}{2(2+4n)}+\frac{4hrana^3T^3}{(4n+1)(4n+2)}+brhT-\frac{2rhbnT}{2n+1}-rhb\alpha T^2+\frac{3bhnr\alpha T^2}{(n+1)(3n+1)}+\frac{3hrb\alpha T^2}{2(n+1)}-\frac{4hrb\alpha^2T^3}{3(n+1)}+\frac{4bhr\alpha^2T^3}{(1+n)(4n+1)}+\frac{2rhb\alpha^2T^3}{(2+4n)}-\frac{4hbrn\alpha^2T^3}{(4n+1)(4n+2)}-\frac{4bhr\alpha^3T^3}{3(2+4n)}+\frac{5rhbn\alpha^3T^4}{(4n+2)(5n+1)}+\frac{C_1\alpha r}{n+1}+\frac{2C_1r\alpha^2T}{4n+2}-\frac{K}{T^2}=0 \tag{9}$$

$$\frac{\delta^2(TC)}{\delta T^2}=-\frac{2rana}{n+1}+\frac{2rana}{2n+1}+\frac{2ra\alpha}{n+1}-\frac{6raT\alpha^2}{2(n+1)}+\frac{6ranT\alpha^2}{(n+1)(3n+1)}+\frac{6raT\alpha^2}{4n+2}-\frac{6raTn\alpha^2}{(3n+1)(4n+3)}-\frac{6raT\alpha^3}{2(4n+2)}+\frac{12nraT^2\alpha^3}{(4n+1)(4n+2)}+br-\frac{2rbn}{2n+1}-2brT\alpha+\frac{6bnrT\alpha}{(n+1)(3n+1)}+\frac{6rbT\alpha}{2(n+1)}-\frac{4rbT^2\alpha^2}{n+1}+\frac{12brT^2\alpha^2}{(1+n)(4n+1)}+\frac{6rbT^2\alpha^2}{4n+2}-\frac{12brn\alpha^2T^2}{(4n+1)(4n+2)}-\frac{4br\alpha^3T^2}{4n+2}+\frac{20rbnT^3\alpha^3}{(2+4n)(5n+1)}+\frac{2C_1r\alpha^2}{4n+2}+\frac{K}{T^3}>0 \tag{10}$$

Equations (9) can be solved to obtain the minimum value of T and then from equations (3) and (7), the optimal value of TC and Q can be evaluated respectively.

4. NUMERICAL EXAMPLES

The proposed model is illustrated by considering some numerical example in this section. In this example, we evaluated the solution of the inventory problem; calculate the schedule period and ordering quantity level.

Example: Considering the following parametric values for the inventory model:

K = 50 unit/year, a= 0.5, b = 0.01, C_1 = ₦1.5/ units, r = 100 units/year, n = 0.5, $\alpha = 0.4$ h = 2.5

Taking into account these numerical values, from equation (9) we obtain a non-linear equation. Using Maple software 2018, to solve the equation, we obtain schedule period for which the inventory is zero to be 0.561687 year, replenishment order quantity Q is 116.240289 units, inventory total cost TC is ₦168.187073

From equation (10), the second partial derivative of the cost function TC with respect to the variable T is positive, which satisfied the sufficient condition i.e. $\frac{\delta^2TC(T)}{\delta T^2}=2.102696>0$, establishing that it is the minimum point and the implication of it is that the model is correct and efficient.

Fig.2 shows that the function TC is convex with respect to T (Schedule period).

5. Sensitivity Analysis

The effect of changes in the values of various parameters r, h, K, C, n, a, b and α are observed in this section on the optimum total cost and the optimum order quantity. The sensitivity analysis is carried out by changing each of the parameters by +20%, +10%, -10%,-20% taking one parameter at a time and keeping other parameters constant. The analyses are based on the example above and the results are displayed in the Table 1.

The following observations are derived from the sensitivity analysis.

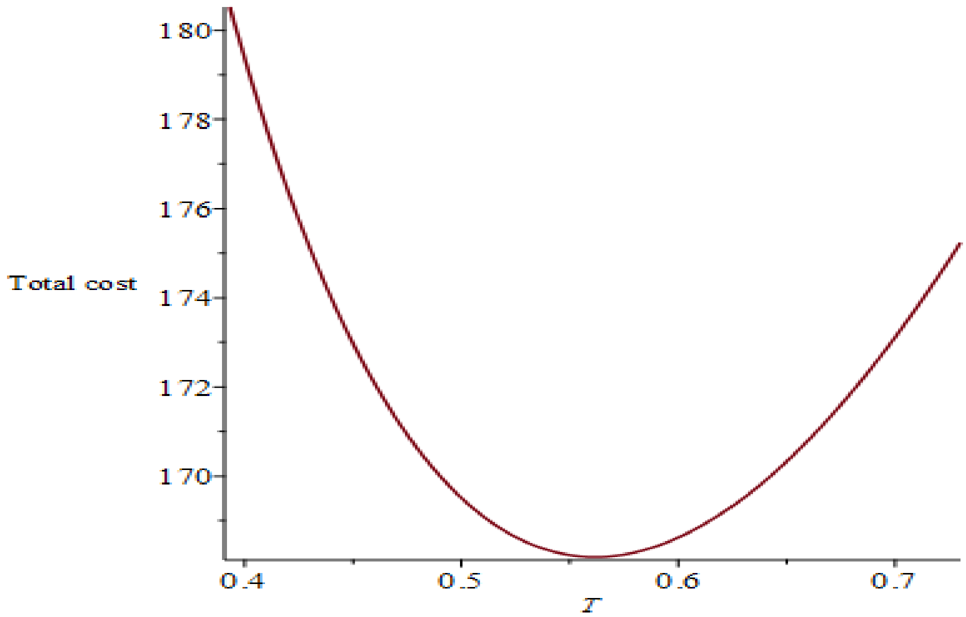
- T decreases while TC and Q increases with the increase in the value of the parameter r, and here T, TC, and Q are highly sensitive to changes in r.
- T decreases while TC and Q increase with increase in the parameter α. Here T, TC and Q are moderately sensitive to change in α.
- T, TC and Q increases with increase in the value parameter K. It could be observed here that T, TC and Q are moderately sensitive to change in K.
- As the value of parameter C and H, increases, there is decrease in T and Q with increase in TC. Here, T, TC and Q have low sensitivity to change in C and h.
- T increases while TC and Q decrease with increase in the value of the parameter n. Here TC, T and Q are very insensitive to changes in n.
- TC increase while T and Q decrease with the increase in the value of parameter a and b. We can deduct from here that T and TC is moderately sensitive to change in a and b, but have a low sensitivity to change in Q.
- Parameter b is very insensitive to change in T, TC, and Q.

Table 1: Sensitivity Analysis for various Inventory Parameters

Parameter	Change %	T	TC	Q
120	+20	0.517285	183.291379	137.837522
110	+10	0.538055	175.913151	127.056761
r = 100	0	0.561687	168.187073	116.240289
90	-10	0.588922	160.058916	105.382713
80	-20	0.620803	151.458742	94.477066
0.48	+20	0.537855	174.810617	118.87766
0.44	+10	0.549365	171.532519	117.575428
α = 0.40	0	0.561687	168.187073	116.240289
0.36	-10	0.574924	164.769462	114.869118
0.32	-20	0.589199	161.274213	113.458316
60	+20	0.609578	185.259683	117.741755
55	+10	0.586273	176.897771	117.00881
K = 50	0	0.561687	168.187073	116.240289
45	-10	0.535617	159.074335	115.430662
40	-20	0.507799	149.491078	114.572746
1.80	+20	0.547338	172.991309	115.793996
1.65	+10	0.554373	170.605924	116.012598
C_1 =1.5	0	0.561687	168.187073	116.240289
1.35	-10	0.569297	165.733352	116.477649
1.20	-20	0.577223	163.243259	116.725359
3.0	+20	0.529280	178.777145	115.234683
2.75	+10	0.544714	173.571848	115.712560
h= 2.5	0	0.561687	168.187073	116.240289
2.25	-10	0.580482	162.602712	116.827357
2.0	-20	0.601462	156.794574	117.486013
0.6	+20	0.583558	162.281344	115.827277
0.55	+10	0.572582	168.223627	116.025263
n=0.50	0	0.561687	168.187073	116.240289
0.45	-10	0.550861	171.315899	116.473841
0.40	-20	0.540091	174.575505	116.727608
0.60	+20	0.529491	178.737796	115.241203
0.55	+10	0.544833	173.550940	115.716252
a =0.50	0	0.561687	168.187073	116.240289
0.45	-10	0.580328	162.626638	116.822536
0.40	-20	0.601104	156.846176	117.474744

0.012	+20	0.561418	168.231674	116.231907
0.011	+10	0.561552	168.209379	116.236083
b= 0.010	0	0.561687	168.187073	116.240289
0.009	-10	0.561822	168.164755	116.244496
0.008	-20	0.561957	168.142425	116.248703

Figure 2: The total cost with respect to T



6. Conclusions

This paper considered a deteriorating inventory model with constant deterioration rate and the demand rate follows a power pattern. Shortages are not allowed in the model and holding cost is assumed to be a linear function of time. Power demand pattern rate is chosen because of the new products in the market now in which the demand is dependent upon time. When they are introducing in the market, the demand may be constant for some time after which the products will gain recognitions, and then the demand of such products increases. Examples of such products are android phones, fashions, electronics, computers etc.

Holding cost is assumed to be dependent on time because it has multiple factors which can allowed it to be represented as a linear function of time.

The results from Tab. 1 show that there is an increase in the total cost as the holding cost is increasing thereby reducing the schedule period and the ordering quantity. It is also discovered that as the deterioration rate is increasing, there is increase in the total cost, leading to increase in the order quantity and reducing the scheduling period.

The model can be useful in the control of inventory of business enterprises that deal with products that have its demand and holding cost dependent on time.

The paper can be extended in many ways, for example, shortages can be introduce into the model, another direction is to consider preservative technology and price dependent demand.

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