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An M/M/1 Retrial Queue With Working Vacation And Non-persistent Customers Under Bernoulli Schedule

[1]*Jinping Xu*, [2]*Juntong Li*, [3]*Tao Li*[+], [4]*Zhenzhen Xu*

[123]*School of Mathematics and Statistics, Shandong University of Technology, Zibo, 255049, Shandong, China*

[4]*Weifang Medical University, Weifang, 261041, Shandong, China*

Abstract: In this paper, we considered an M/M/1 retrial queue with working vacation and non-persistent customers under Bernoulli schedule. Request retrials from the orbit of infinite size follow a Poisson process with rate α. When the system is empty, the server either takes an ordinary vacation with probability $p(0 \leq p \leq 1)$ or a working vacation with probability $\bar{p}(\ \bar{p} = 1 - p\ )$. Using the matrix-analytic method, the stationary probability distribution is obtained on the stability condition. Finally, showing the effect of the models parameters on the system's characteristics by some numerical examples.

Key words: Retrial; Working vacation; Ordinary vacation; Bernoulli.

1 Introduction

In the queueing systems, the single working vacation is an important type of queueing model. Servi and Finn [1] first introduced the working vacation policy and researched an M/M/1 queue. During a vacation, the server commits a lower service rather than completely stopping the service. If the service is reduced to zero in a working vacation, the queue with working vacation will become an ordinary vacation. Therefore, the queue with working vacation is a promotion of the queue with ordinary vacation. Then onwards, several articles [2-6] appeared and analyzed the single server queue with single working vacation or multiple working vacations.

Retrial queueing systems are described by characteristics that the arriving customers who find the server busy enter the retrial orbit to try again their requests. Recently, the retrial queueing systems with working vacation have been investigated extensively. Do [7] first analyzed an M/M/1 queue with retrials and working vacations, and Yi [8] discussed the discrete-time Geo/Geo/1 retrial queue with working vacations. Another feature which has been widely discussed in retrial queueing systems is the non-persistent (or impatient) customer. Moreover, retrial queues with impatient customers have attracted more attentions. Li [9] treated an M/M/1 retrial G-queue with Bernoulli working vacation interruption and non- persistent customers, where the retrial customer leave the system or go back to the retrial orbit if the retrial customer becomes non- persistent. Gao and Wang [10] studied an M/G/1-G retrial queues with orbital search and non-persistent customers. In this paper, we mainly analyze an M/M/1 retrial queue with Bernoulli-Schedule-Control- led vacation and non-persistent.

This paper is organized as follows. In section 2, we establish the model and obtain the infinitesimal generator. In section 3, we get the minimal non-negative solution and the stationary probability distribution under the stability condition. Some important performance indexes are also derived. In section 4, some numerical examples are given to illustrate the influence of parameters on the model. Finally, section 5 concludes this paper.

2 Model Formulation

In this paper, we consider an M/M/1 retrial queue with working vacation and non-persistent customers under Bernoulli schedule. The detailed description of this model is given as follows:

(1) The customers arrive at the system in accordance with a Poisson process with rate λ. If the arriving customer finds the server is idle, the customer accepts the service immediately. On the other hand, if the server is busy, the customer is forced to wait in the orbit of infinite size.

(2) Request retrials from the orbit follow a Poisson process with rate α. When the customer retries, if the server is found to be free, customer gets service immediately. If the server is found to be busy, on the other hand, the retrial customer becomes non-persistent (leaves the system) with probability $q(0 \leq q \leq 1)$ or returns into the orbit with probability $\bar{q}(\bar{q} = 1 - q)$.

(3) During a normal service busy period, the service times S_b is governed by an exponential distribution with parameter μ. In a working vacation busy period, the service times S_v follows an exponential distribution with parameter ν.

(4) When the system becomes empty, the server begins an normal vacation with probability $p(0 \leq p \leq 1)$ or a working vacation with probability $\bar{p}(\bar{p} = 1 - p)$.

(5) When an ordinary vacation or a working vacation ends, the server will go to the normal period. The working vacation time follows an exponential with parameter η. And the ordinary vacation time follows an exponential with parameter θ.

We suppose that the inter-arrival times, the inter-retrial times, the service times during a normal busy period, the service times during a working vacation busy period, the ordinary vacation times and the working vacation times are mutually independent.

Let $Q(t)$ be the number of customers in the orbit at time t, and $J(t)$ be the state of server at time t. There are five possible states of the server as follows:

$$J(t) = \begin{cases} 0\text{, the server is during an ordinary vacation period at time } t, \\ 1\text{, the server is in a working vacation period at time } t \text{ and the server is free}, \\ 2\text{, the server is in a working vacation period at time } t \text{ and the server is busy}, \\ 3\text{, the server is during a normal service period at time } t \text{ and the server is free}, \\ 4\text{, the server is during a normal service period at time } t \text{ and the server is busy}. \end{cases}$$

Obviously, $\{Q(t), J(t), t \geq 0\}$ is a quasi-birth and death process, we refer to it simply as QBD process. Their state space can be expressed as

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$$\Omega = \{(k, j), k \geq 0, j = 0,1,2,3,4\}$$

Using the lexicographical sequence for the states, the infinitesimal generator can be written as

$$Q = \begin{bmatrix} B_0 & A & & & \\ C & B & A & & \\ & C & B & A & \\ & & \ddots & \ddots & \ddots \end{bmatrix}$$

where,

$$B_0 = \begin{bmatrix} -(\lambda+\theta) & 0 & 0 & \theta & 0 \\ 0 & -(\lambda+\eta) & \lambda & \eta & 0 \\ 0 & \nu & -(\lambda+\eta+\nu) & 0 & \eta \\ 0 & 0 & 0 & -\lambda & \lambda \\ p\mu & \bar{p}\mu & 0 & 0 & -(\lambda+\mu) \end{bmatrix},$$

$$B = \begin{bmatrix} -(\lambda+\theta) & 0 & 0 & \theta & 0 \\ 0 & -(\lambda+\eta+\alpha) & \lambda & \eta & 0 \\ 0 & \nu & -(\lambda+\eta+\nu+q\alpha) & 0 & \eta \\ 0 & 0 & 0 & -(\lambda+\alpha) & \lambda \\ 0 & 0 & 0 & \mu & -(\lambda+\mu) \end{bmatrix},$$

$$A = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & q\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

In the process of quasi birth and death analysis, the matrix equation satisfies

$$R^2B + RB + A = 0 \tag{2.1}$$

Its smallest nonnegative solution is called a rate matrix, expressed in terms of R.

3 Stability Condition and Stationary Distribution

Theorem 3.1 The QBD process $\{Q(t), J(t)\}$ is positive recurrent if and only if $(\mu - \lambda)\alpha > \lambda^2$.

Proof. The above inequality follows easily from: Firstly, we assume that

$$G = A + B + C = \begin{bmatrix} -\theta & 0 & 0 & \theta & 0 \\ 0 & -(\lambda+\eta+\alpha) & \lambda+\alpha & \eta & 0 \\ 0 & \nu & -(\eta+\nu) & 0 & \eta \\ 0 & 0 & 0 & -(\lambda+\alpha) & \lambda+\alpha \\ 0 & 0 & 0 & \mu & -\mu \end{bmatrix},$$

According to the Theorem 7.3.1 in [11], if the matrix G is irreducible, we can get the condition for positive recurrence of the QBD. After permutation of rows and columns, the QBD is positive recurrent if and only if

$$\pi \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} e > \pi \begin{bmatrix} 0 & 0 \\ 0 & \lambda \end{bmatrix} e,$$

Where e is a column vector with all elements equal to one, and π satisfies the next equations

$$\pi \begin{bmatrix} -(\lambda+\alpha) & \lambda+\alpha \\ \mu & -\mu \end{bmatrix} = 0, \pi e = 1.$$

By simple mathematical operations, the QBD process is positive recurrent if and only if $(\mu-\lambda)\alpha > \lambda^2$.

Theorem 3.2 If $(\mu-\lambda)\alpha > \lambda^2$, the matrix equation (2.1) has the minimal non-negative solution

$$R = \begin{bmatrix} r_1 & 0 & 0 & r_4 & r_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & r_2 & r_3 & r_6 & r_7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_8 & r_9 \end{bmatrix},$$

where

$$r_1 = \frac{\lambda}{\lambda+\theta}, r_2 = \frac{\nu}{\lambda+\eta+\alpha} r_3,$$

$$r_3 = \frac{(\lambda+\eta+\nu+q\alpha)(\lambda+\eta+\alpha)-\lambda\nu-\sqrt{(\lambda\nu-(\lambda+\eta+\nu+q\alpha)(\lambda+\eta+\alpha))^2-4\lambda(\alpha\nu+q\alpha)(\lambda+\eta+\alpha)}}{2(\alpha\nu+q\alpha)},$$

$$r_4 = \frac{\lambda}{\alpha}, r_5 = \frac{\lambda^2(\lambda+\alpha+\theta)}{\alpha\mu(\lambda+\theta)},$$

$$r_6 = \frac{r_2\eta+r_3\eta}{\alpha(1-r_3)}, r_7 = \frac{r_6(\lambda+\alpha r_3)+r_3\eta}{\mu},$$

$$r_8 = \frac{\lambda}{\alpha}, r_9 = \frac{\lambda(\lambda+\alpha)}{\alpha\mu}.$$

Proof. Because block matrices A, B, C are upper triangular matrix, we can assume

$$R = \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix}$$

there R_{11} is a 3×3 matrix, R_{12} is a 3×2 matrix and R_{22} is a 2×2 matrix. R be substituted into the above equation matrix, we can get R_{11}, R_{12}, R_{22} by some calculations.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = {R_{11}}^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha \\ 0 & 0 & q\alpha \end{bmatrix} + R_{11} \begin{bmatrix} -(\lambda+\theta) & 0 & \theta \\ 0 & -(\lambda+\eta+\alpha) & \lambda \\ 0 & \nu & -(\lambda+\eta+\nu+q\alpha) \end{bmatrix} + \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = (R_{11}R_{12} + R_{12}R_{22}) \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} + R_{11} \begin{bmatrix} \theta & 0 \\ \eta & 0 \\ 0 & \eta \end{bmatrix} + R_{12} \begin{bmatrix} -(\lambda+\alpha) & \lambda \\ \mu & -(\lambda+\mu) \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = {R_{22}}^2 \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} + R_{22} \begin{bmatrix} -(\lambda+\alpha) & \lambda \\ \mu & -(\lambda+\mu) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \lambda \end{bmatrix}.$$

From the first matrix equation, we get $R_{11} = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & r_2 & r_3 \end{bmatrix}$. $R_{22} = \begin{bmatrix} r_4 & r_5 \\ 0 & 0 \\ r_6 & r_7 \end{bmatrix}$ can be obtained by the third matrix equation. Taking R_{11} and R_{22} into the second matrix equation, we finally gain $R_{12} = \begin{bmatrix} 0 & 0 \\ r_8 & r_9 \end{bmatrix}$.

Under the stability condition, let (Q, J) be the stationary limit of the process {Q(t), J(t)} and denote

$$\pi_k = (\pi_{k0}, \pi_{k1}, \pi_{k2}, \pi_{k3}, \pi_{k4}), k \geq 0$$

$$\pi_{kj} = P(Q = k, J = j) = \lim_{t\to\infty} P\{Q(t) = k, J(t) = j\}, (k, j) \in \Omega.$$

Theorem 3.3 If $(\mu - \lambda)\alpha > \lambda^2$, the stationary probability distribution of (Q, J) is given by

$$\begin{cases} \pi_{k0} = \pi_{00}{r_1}^k, & k \geq 1 \\ \pi_{k1} = \pi_{02}r_2{r_3}^{k-1}, & k \geq 1 \\ \pi_{k2} = \pi_{02}{r_3}^k, & k \geq 1 \\ \pi_{k3} = \pi_{00}\left[r_4{r_1}^{k-1} + \frac{r_5r_8}{r_9-r_1}\left({r_9}^{k-1} - {r_1}^{k-1}\right)\right] & \\ \quad +\pi_{02}\left[r_6{r_3}^{k-1} + \frac{r_7r_8}{r_9-r_3}\left({r_9}^{k-1} - {r_3}^{k-1}\right)\right] + \pi_{04}r_8{r_9}^{k-1}, & k \geq 1 \\ \pi_{k4} = \pi_{00}\frac{r_5}{r_9-r_1}\left({r_9}^k - {r_1}^k\right) + \pi_{02}\frac{r_7}{r_9-r_3}\left({r_9}^k - {r_3}^k\right) + \pi_{04}{r_9}^k, & k \geq 1 \end{cases} \tag{3.1}$$

and

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$$\begin{cases} \pi_{00} = \left(\frac{p(\lambda+\eta)}{\bar{p}(\lambda+\theta)} - \frac{p\lambda\nu}{\bar{p}(\lambda+\theta)(\lambda+\eta+\nu-\alpha r_2-q\alpha r_3)}\right)\pi_{01}, \\ \pi_{02} = \frac{\lambda}{\lambda+\eta+\nu-\alpha r_2-q\alpha r_3}\pi_{01}, \\ \pi_{03} = \left(\frac{\eta}{\lambda} + \frac{p\theta(\lambda+\eta)}{\bar{p}\lambda(\lambda+\theta)} - \frac{p\theta\lambda\nu}{\bar{p}\lambda(\lambda+\theta)(\lambda+\eta+\nu-\alpha r_2-q\alpha r_3)}\right)\pi_{01}, \\ \pi_{04} = \left(\frac{\lambda+\eta}{\bar{p}\mu} - \frac{\lambda\nu}{\bar{p}\mu(\lambda+\eta+\nu-\alpha r_2-q\alpha r_3)}\right)\pi_{01}. \end{cases} \quad (3.2)$$

Proof. Using the matrix-geometric solution method, we have

$$\pi_k = (\pi_{k0}, \pi_{k1}, \pi_{k2}, \pi_{k3}, \pi_{k4}) = \pi_{k0}R^k = (\pi_{00}, \pi_{01}, \pi_{02}, \pi_{03}, \pi_{04})R^k, k \geq 0$$

Because

$$R^k = \begin{bmatrix} {r_1}^k & 0 & 0 & r_4{r_1}^{k-1} + \frac{r_5 r_8}{r_9 - r_1}\left({r_9}^{k-1} - {r_1}^{k-1}\right) & \frac{r_5}{r_9 - r_1}\left({r_9}^k - {r_1}^k\right) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & r_2{r_3}^{k-1} & {r_3}^k & r_6{r_3}^{k-1} + \frac{r_7 r_8}{r_9 - r_3}\left({r_9}^{k-1} - {r_3}^{k-1}\right) & \frac{r_7}{r_9 - r_3}\left({r_9}^k - {r_3}^k\right) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_8{r_9}^{k-1} & {r_9}^k \end{bmatrix},$$

taking R^k into the above equation, we can obtain (3.1). Besides, π_0 satisfies the next equation

$$\pi_0 B_0 + RC = 0 \quad (3.3)$$

Using equation (3.3), we can get (3.2) by some calculations. Due to

$$\sum_{j=0}^{4}\sum_{k=0}^{\infty}\pi_{kj} = 1$$

we can obtain

$$\pi_{01} = (1 + a + b + c + d)^{-1}$$

where

$$a = \frac{(1 + r_6 + r_2)(1 - r_9) + r_7(1 + r_8)}{(1 - r_3)(1 - r_9)} \times \frac{\lambda}{\lambda + \eta + \nu - \alpha r_2 - q\alpha r_3},$$

$$b = \frac{(1 + r_4)(1 - r_9) + r_5(1 + r_8)}{(1 - r_1)(1 - r_9)} \times \left(\frac{p(\lambda + \eta)}{\bar{p}(\lambda + \theta)} - \frac{p\lambda\nu}{\bar{p}(\lambda + \theta)(\lambda + \eta + \nu - \alpha r_2 - q\alpha r_3)}\right),$$

$$c = \frac{1 + r_8}{1 - r_9} \times \left(\frac{\lambda + \eta}{\bar{p}\mu} - \frac{\lambda\nu}{\bar{p}\mu(\lambda + \eta + \nu - \alpha r_2 - q\alpha r_3)}\right),$$

$d = \frac{\eta}{\lambda} + \frac{p\theta(\lambda+\eta)}{\bar{p}\lambda(\lambda+\theta)} - \frac{p\theta\lambda\nu}{\bar{p}\lambda(\lambda+\theta)(\lambda+\eta+\nu-\alpha r_2-q\alpha r_3)}$.

Obviously, the state probability of the server is written by

$$P_0 = P\{J=0\} = \sum_{k=0}^{\infty} \pi_{k0} = \frac{1}{1-r_1}\pi_{00},$$

$$P_1 = P\{J=1\} = \sum_{k=0}^{\infty} \pi_{k1} = \frac{r_2}{1-r_3}\pi_{02} + \pi_{01},$$

$$P_2 = P\{J=2\} = \sum_{k=0}^{\infty} \pi_{k2} = \frac{1}{1-r_3}\pi_{02},$$

$$P_3 = P\{J=3\} = \sum_{k=0}^{\infty} \pi_{k3} = \pi_{03} + \frac{r_4(1-r_9)+r_5r_8}{(1-r_1)(1-r_9)}\pi_{00} + \frac{r_6(1-r_9)+r_7r_8}{(1-r_3)(1-r_9)}\pi_{02} + \frac{r_8}{1-r_9}\pi_{04},$$

$$P_4 = P\{J=4\} = \sum_{k=0}^{\infty} \pi_{k4} = \frac{r_5}{(1-r_1)(1-r_9)}\pi_{00} + \frac{r_7}{(1-r_3)(1-r_9)}\pi_{02} + \frac{1}{1-r_9}\pi_{04}.$$

When the server is busy, the probability is

$$P_b = P\{J=2\} + P\{J=4\} = P_2 + P_4.$$

When the server is free, the probability is

$$P_f = P\{J=0\} + P\{J=1\} + P\{J=3\} = P_0 + P_1 + P_3 = 1 - P_b.$$

Letting L_0 be the number of customers in the orbit, we can get

$$\begin{aligned} E[L_0] &= \sum_{k=1}^{\infty} k(\pi_{k0} + \pi_{k1} + \pi_{k2} + \pi_{k3} + \pi_{k4}) \\ &= \frac{(r_1+r_4)(1-r_9)^2 + r_5r_8(2-r_1-r_9) + r_5(1-r_1r_9)}{(1-r_1)^2(1-r_9)^2}\pi_{00} \\ &+ \frac{(r_2+r_3+r_6)(1-r_9)^2 + r_7r_8(2-r_3-r_9) + r_7(1-r_3r_9)}{(1-r_3)^2(1-r_9)^2}\pi_{02} \\ &+ \frac{r_8-r_9}{(1-r_9)^2}\pi_{04} \end{aligned}$$

Letting L be the number of customers in the system, we have

$$E[L] = \sum_{k=1}^{\infty} k(\pi_{k0} + \pi_{k1} + \pi_{k3}) + \sum_{k=0}^{\infty}(k+1)(\pi_{k2} + \pi_{k4}) = E[L_0] + P_2 + P_4 = E[L_0] + P_b$$

The expected waiting time of a customer in the orbit (system) is represented by $E[W_0]$(E[W]) using Little's formula

$$E[W_0] = \frac{E[L_0]}{\lambda}, E[W] = \frac{E[L]}{\lambda}$$

4 Numerical Results

In this part, taking λ = 1, μ = 2, θ = 0.4, the effects of some parameters on the expected queue length E[L] are presented in Figures 1-3. The next are some findings.

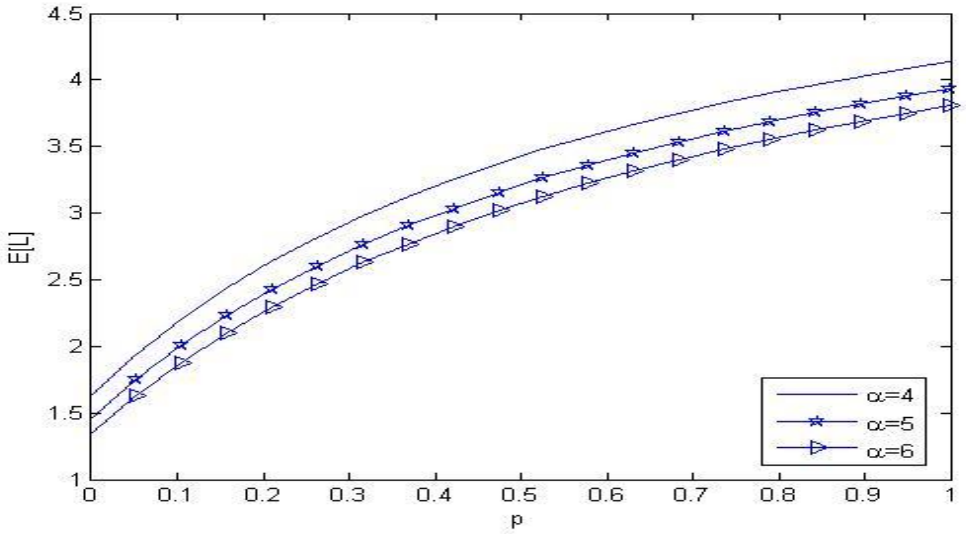

Figure 1: E[L] with change of p

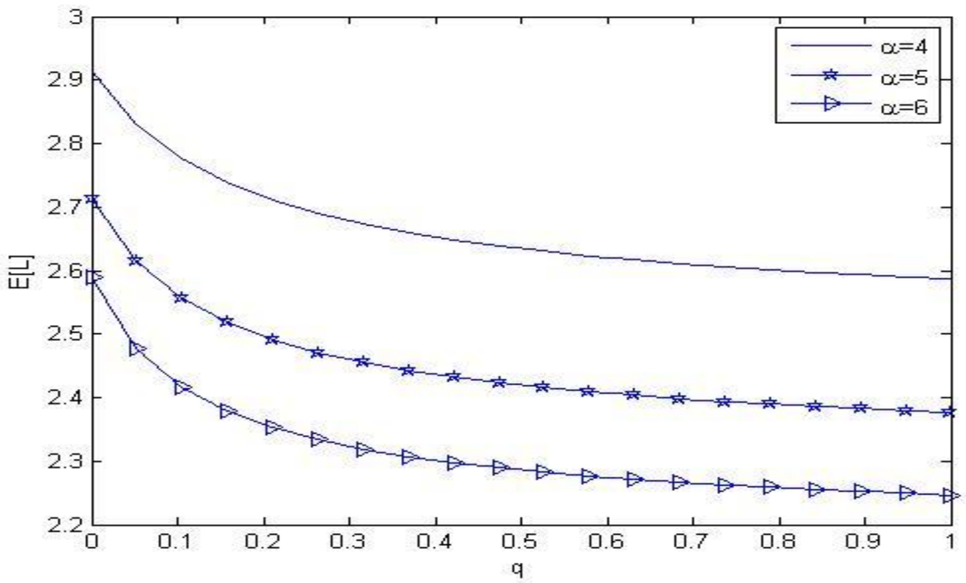

Figure 2: E[L] with change of q

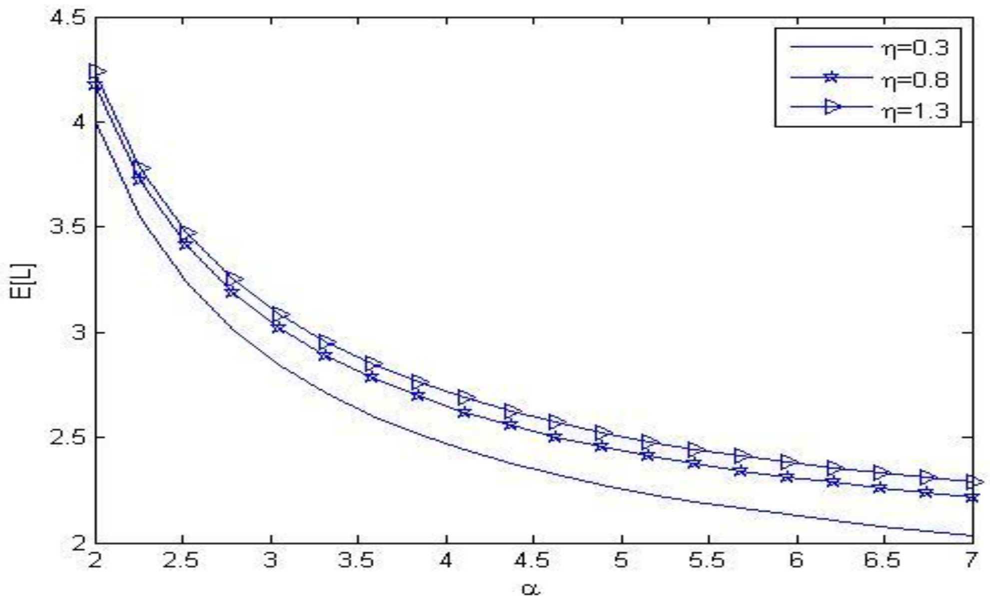

Figure 3: E[L] with change of α

In Figure 1, letting $\nu = 0.5, \eta = 0.8$, $q = 0.8$, we know the effect of probability p on E[L] for different values of α. We discover that E[L] increases with an increasing value of p. This is because that if the server turns in the ordinary vacation, the service will be stopped immediately. In addition, when retrial rate α is lager, E[L] is reduced, and which complies our expectation.

In Figure 2, letting $\nu = 0.5, \eta = 0.8$, $p = 0.2$, we find that E[L] decreases with the probability q increasing. The reason is that the retrial customer leaves the system with probability q when the server is busy.

In Figure 3, letting $\nu = 0.5$, $q = 0.8$, $p = 0.2$, we can see E[L] decreases monotonously as the values of α increases. It is easily explained by the fact that the inter retrial time becomes shorter. Moreover, under the same condition, it is easy to see that E[L] is greater with the probability η is larger.

5 Conclusions

This paper analyzed a single server retrial queue with Bernoulli- schedule-controlled vacation and non-persistent. Using the matrix- analytic method, the stationary probability distribution and some performance measures are obtained. By some numerical examples, we can study the effects of some parameters on the feature of this model.

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